AN INTEGRATED DESIGN DECISION SYSTEM FOR OPTIMUM LIFE-CYCLE COST WITH EMPHASIS ON CONSTRAINT MANAGEMENT

By

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dedicated to my parents

Mr. Ram K. Tandon
and
Mrs. Kusum D. Tandon

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KEY TO SYMBOLS

English Symbols

B	A conversion factor, (1.054 Btu/kWhr).
B	Cost per measurement of the quality characteristic.
b	A constant in lubricant oil viscosity-temperature relation.
b	Energy conversion factor (1.054 Btu/kWhr).
C	Cost per process adjustment in quality control.
C	Bearing clearance (in).
ΔC	Incremental increase in bearing clearance (in) during simulation.
C_{eq}	External quality cost (cost to customers).
C_{iq}	Internal quality cost (in-plant quality cost).
C_i	Initial clearance at the beginning of life (in).
C_f	Final clearance at the end of bearing life (in).
C_f	Cost of part failure (or, cost of unscheduled corrective maintenance including down-time).
C_{lc}	Life-cycle cost.
C_{min}, C_{max}	Minimum and maximum allowable clearance (in).
C_M	Manufacturing cost.
C_m	Cost of replacements of unfailed parts (or, cost of preventive maintenance).
C_o	Operation cost of bearing (\$).
$C_o'', C_o''(C_i)$	Second derivative of operations cost with respect to initial clearance.
C_{oe}	Cost of energy loss in the bearing (\$).
C_{or}	Cost of oil replacement in the bearing (\$).
C_R	Reliability cost.
c_s	Heat capacity of lubricant oil (0.42 Btu/lbf-°F).
D	Bearing diameter (in).
d_p	Average size of contaminant particles (in) in oil.
F^*_i	A set of transformation functions for stage i in dynamic programming.
F_μ	A factor representing the deterioration in viscosity.
f	Coefficient of friction.
$f(X)$	Objective function.
G_μ	Relative (normalized) oil viscosity.
H_c, H_b, H_s	Hardness of contaminant particles, bearing and shaft material respectively.

$H(t)$ Energy loss in bearing due to friction (kW).
h_o Minimum oil film thickness in the bearing (in).
$h_{o,min}$ Lower allowable limit on minimum oil film thickness (in).
$h_{o,max}$ Upper allowable limit on minimum oil film thickness (in).
$h(X)$ Equality constraint function.
$g(X)$ Inequality constraint function.
G_{im} m'th constraint at i'th stage in dynamic programming.
J Mechanical equivalent of heat (9336 lbf-in/Btu).
J_i Number of transformation equations (and output state variables) for stage i in dynamic programming.
K A constant used to evaluate the thermo-hydrodynamic equilibrium point.
K_b,K_s Wear coefficients for bearing and shaft material respectively.
k_o-k_3 Manufacturing cost coefficient.
L Bearing length (in).
MTBR Mean time between replacements.
MTTF Mean time between failures.
N Total number of parts replacement during life-cycle (N_f+N_m).
N Bearing speed, (rps).
N_f Expected number of failures (or, number of unscheduled corrective maintenance) during the life-cycle.
N_i Total number of design variables at i'th stage of the dynamic programming.
N_m Number of replacements of unfailed parts (or, number of preventive maintenance) during the life-cycle.
n Quality measurement interval, i.e. measurements made at every n product during quality control.
n Number of bearing revolutions since last oil replacement.
P_{max} Maximum value of pressure in oil film (psi).
$P_{max,u}$ Maximum value of pressure in oil film (psi).
Q Oil flow rate through the bearing (in^3/sec).
Q_i Sum of the objective function for stage i and the optimum value function for stage i+1 in dynamic programming.
R Radius of the bearing.
$R_i(S_{i-1})$ Optimal value function for stage i in dynamic programming.
$R(t)$ Reliability of a component as a function of its operation time.
$R_M(t)$ Reliability of a maintained component (with preventive maintenance) as a function of its operation time.
r_{co} Ratio of contaminants in oil by volume.
r_h Ratio of minimum oil film thickness to average contaminants particle size (h_o/d_p).
r_e Energy cost rate (0.01 \$/kWhr).
r_o Cost rate for oil (0.04 \$/in^3).
r_p Multipliers to penalty function.
S Sommerfeld number.

S_{i-1}	State (or input) variables for stage i in dynamic programming.
T_o	Oil outlet temperature.
T_{max}	Maximum allowable oil outlet temperature (°F).
ΔT	Temperature difference between oil inlet and outlet (°F).
t	Operation time (hrs) during bearing simulation.
t_l	Bearing life.
Δt	Operation time step (hrs) used during simulation.
$U_i(\boldsymbol{S}_{i-1},\boldsymbol{X}_i)$	Objective function at stage i in dynamic programming.
u	Predicted average number of products produced between successive adjustment in the process.
V_o	Oil volume to be replaced (in^3), or oil volume being used at one point in time for bearing cooling.
W_o	Specified static load on the bearing (lbf).
W	Dynamic load on the bearing (lbf).
W_v	Bearing wear volume (in^3).
dW_v/dn	Combined wear rate of bearing and shaft by volume, (in^3/revolution).
$\boldsymbol{X,Y,W}$	Decision variable vector.
$\boldsymbol{X}_i$	A set of stage decision variables (or design variables) for stage i in dynamic programming.

Roman Symbols

Δ_D	Design tolerance for the quality characteristics (tolerance on diameter for the bearing) (in).
λ	Dynamic load multiplier.
λ_k	Lagrange multiplier for the k'th constraint.
$\lambda(t)$	Failure rate of a part as a function of its operation time.
μ	Lubricant viscosity (Reyn).
ρ	Oil density (0.0311 lbf/in^3).
ω	Bearing speed (radians/sec).
Θ	A constant in lubricant oil viscosity-temperature relation
$\psi(\sigma)$	Lund and Saible instability criterion
ρ	Oil density (0.0311 lbf/in^3).
σ_o	Standard deviation of the quality characteristic of interest in the manufacturing process.

Abstract of Thesis Presented to the Graduate School
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The objective of this research is to address the need for a comprehensive approach to designing quality products with optimum overall life-cycle cost. Product life-cycle considerations include research and development, design innovations, manufacturing, quality assurance, operations, support, maintenance and waste disposal.

The emphasis in this research is on developing an algorithmic framework for integrating the many separate efforts that deal with each phase of the product life-cycle, in order to design, manufacture, operate and maintain the product with minimum life-cycle cost. A further objective is to develop a framework for

quantitative determination of the value added from research and development, and design innovations. This framework can aid in decision making for investment in research and development and acquiring new technology by performing cost/benefit analysis.

To illustrate the approach an integrated decision system for the optimum design of automotive journal bearings has been developed. The stages in product development, manufacture and operation of the system are serial in nature; hence, a multilevel and dynamic programming approach combined with nonlinear optimization techniques are used for overall optimization. Sensitivity of the optimum solution to problem parameters (design variables and design constraints) is evaluated in order to perform cost/benefit analysis of investments in R&D and new concepts.

CHAPTER 1
INTRODUCTION

1.1 Background

Due to the globally competitive nature of today's manufacturing industries and rising costs of products, it is imperative to design products which are economical to produce and operate. If the products are designed only for optimal performance, their manufacturing, operations and support costs may not be optimum. There is a growing need to design products and systems in terms of optimum life-cycle costs, rather than just a segment of its life-cycle costs such as production costs. Designing the products and systems in terms of optimum life-cycle costs ensures an overall economy and not the economy of purchase price only. Also, in many industries it has been recognized that the greatest impact on total life-cycle cost results from the design decisions made at the early stages of the design of a product. For example, the material specifications or allocation of tolerances at design stage can have great impact on the cost of manufacturing, quality, reliability and life of the product. Thus life-cycle cost is of paramount importance in the decision making process and it must be considered as a major evaluation criterion at the design stage. The life-cycle cost directly or indirectly

accounts for other parameters such as product performance, effectiveness, size, weight, capacity, manufacturability, reliability, maintainability and so on.

Before a design engineer can accommodate the constraints and cost of manufacturing, quality assurance, operation and support of the product, information must be supplied to the designer in a manner that can be readily used. The design engineer, then, not only can evaluate the performance of the product, but also can accommodate the constraints imposed by other functions (departments) of an organization. The components of the life cycle cost can be expressed as a function of various parameters and decision variables which affect its life-cycle.

Often, the minimum life cycle cost may be outside the design constraints. For example, a designer may find that a product design is constrained by minimum surface roughness producible by the existing processes and machines. In the designer's opinion a better surface roughness might lower the life-cycle cost of the product in terms of decreased quality cost and increased useful life. In order to improve the surface roughness of the design, a cost/benefit analysis should be performed to analyze the financial resource allocation in either buying new machines and R&D to develop new processes or analytical theories. A sound methodology for performing such a cost/benefit analysis for expanding the design space should be available to the designer in order to explore the potential gains in the optimum life-cycle cost and consequently to justify any new investments. The development of such a methodology is also one of the goals of this research. The methodology integrates the scientific and management aspects of the product realization.

1.2 Relevant Literature

In spite of the considerable interest and practical importance, the development of automated systems which integrate all aspects of the design, manufacturing, quality assurance, and operation aspects along with the cost management has not received appropriate attention.

1.2.1 Optimum Parameter Design for Performance

Corser and Seireg [Cors85] developed an algorithmic framework for integration of design, manufacturing and inspection stages of the life cycle. An automated design procedure was developed using simplified cost models of manufacturing and quality control functions that reflect the overall cost patterns as a function of design parameters. A simple hydrodynamic bearing was used to illustrate the approach. A dynamic programming method was used to optimize the overall product cost.

Many other systems have been developed to optimize the individual departmental tasks such as design of a mechanical element. Seireg and Ezzat [Seir69] developed an automated design system for the selection of length, clearance, and lubricant viscosity to optimize a performance characteristic of hydrodynamic journal bearings. The objective was to penalize high temperatures and pressures in the designed bearing. Sutherland and Roth [Suth75] considered manufacturing tolerances and costs in the design of mechanisms for function generations problems.

1.2.2 Optimum Inspection Schemes

Many schemes have been presented for optimal inspection and quality control during production. After a design is handed over to the manufacturing department, these schemes deal with the optimal inspection strategy during production in order to conform to the prescribed design as closely as possible. The objective here is to determine the inspection strategy which maximizes the average outgoing quality and minimizes the cost of inspection. These schemes try to achieve a balance between the cost of inspection and the cost of outgoing (external) quality. But these inspection schemes do not try to influence the product specifications at the design and development stage.

A skip-lot sampling plan initially calls for inspection of every lot (normal inspection) and inspection of only a fraction of the lots (skip-lot inspection) when lot acceptance demonstrates the quality to be good. This results in reduction of average number of items inspected while maintaining a probability of acceptance at least as high as lot-by-lot inspection would yield. The important decision parameters here are the sample size inspected, and the acceptance number. Other skip-lot parameters are the number of good lots before skipping is introduced, and the fraction of lots to be inspected during skip lot sampling. Hsu presented an optimal scheme for inspection of skip-lot sampling plans for simple processes [Hsu80]. He also presented an optimal scheme for skip-lot inspection of multistage production processes [Hsu84]. The objective was to minimize the average production, appraisal, and failure cost per unit produced.

X-bar ($\overline{X}$) charts are used during production for process control where the quality characteristic of interest is a continuous variable. In the installation of an $\overline{X}$ control chart monitoring system, the user must specify the four quantities: the sample size n, the time interval (in hours) between samples h, the average value of quality control characteristic $\overline{x}$, and the width of the control limit k (quality control limits = $\overline{x} \pm k\sigma$). Many researchers have presented schemes for optimum economic design of $\overline{X}$ control charts. Montgomery [Mont82a] presented a scheme to choose the three parameters (h,n,k) such that expected loss per hour is minimized. Collani [Coll86] presented a scheme to choose these parameters so that expected loss per item is minimized.

Other inspection and quality control optimization studies for different sampling plans have been reported in the literature. Some of the important studies include Hassan and Knowles [Hass79]. Gibra [Gibr78, Gibr81] presented models for economical optimal determination of the parameters of np-control charts. Collani and Sheil presented an approach for controlling the process variability [Coll89]. They gave an economically optimal method of controlling a process using $\overline{X}$ chart and s chart so as to maximize the average profit per item produced. The method determines the optimal values of sampling interval h, sample size n, and parameter k. The process is considered out of control whenever a plotted point on the s chart exceeds the limit $k\sigma_o$. Montgomery [Mont2] presents an excellent overview and literature survey of economic design of control charts.

1.2.3 Statistical Tolerance Analysis

Many quality engineering studies deal with the techniques for statistical tolerancing. Evans presents an excellent description of the state of the art in tolerancing [Evan74, Evan75a, Evan75b]. The statistical tolerancing is generally taken to mean the following two aspects: (1) the problem of ascertaining the distribution of the response of a mechanism, when the component tolerance distributions are given, and (2) the problem due to shifting and drifting of component tolerance distributions. These references do not deal with the problem of determining the component tolerances, given the overall tolerance on the performance characteristics of the system.

An implementation aspect of tolerancing is the *tolerance allocation*, where the task is to find the component tolerances, given the assembly tolerance. Chase and Greenwood [Chas88] presented a tolerance allocation method which minimizes the cost of production or assembly. This is accomplished by defining a manufacturing cost versus tolerance curve for each component in the assembly. The optimization algorithm varies the tolerance for each component and searches for the best combination of tolerances that minimizes the manufacturing cost. Taguchi's quality engineering methods include optimum tolerance design as presented in the following section.

1.2.4 Taguchi Methods of Quality Engineering

Genichi Taguchi's approach to quality involves engineering and statistical methods to achieve improvements in cost and quality by optimizing the product design and manufacturing process. Taguchi defines two aspects of an overall quality

engineering system — on-line and off-line quality control [Tagu78a]. The quality control activities at the manufacturing stage, which are conducted to keep the manufacturing process in statistical control and to reduce the manufacturing variations (imperfections) in the product, are termed *on-line quality control* methods. Quality control methods such as cause and effect diagrams, process capability studies, process quality control, and control charts are referred to by Taguchi as on-line quality control methods. *Off-line quality control* methods are quality control and activities conducted at the product or process design stage in the product development cycle. The overall aim of the off-line quality control activities is to improve product reliability, manufacturability, and to reduce product development and product operation costs. Off-line quality activities include design reviews, sensitivity analysis, prototype tests, accelerated life tests, and reliability studies.

Taguchi also formalized a three-step process for off-line quality engineering [Tagu86, Tagu89] which includes system design, parameter design, and tolerance design. *System design* is a process of applying scientific and engineering knowledge to produce a basic functional prototype design. The system design defines the initial settings of product or process design characteristics. Once the system design is accomplished, *parameter design* is undertaken to identify and establish the optimum values of various parameters (nominal settings) that minimize the performance variation in the system. The variations in performance characteristics from its optimal values are generally a function of the design parameters.

After optimum parameter design for a product, an important and often critical task faced by a designer is *tolerance design* — determining the manufacturing tolerances around the nominal settings for various parameters. Tolerance design is a method for determining tolerances that minimize the sum of product manufacturing cost, and cost of product operation over its lifetime. Tolerances that are too narrow increase the manufacturing cost, and tolerances that are too wide increase the performance variation and, therefore, a product's lifetime operation cost.

Taguchi's tolerance design methods are based on minimizing the manufacturing and operational costs. He proposed relationships between tolerances, quality control parameters and manufacturing capability by using quadratic functions to model cost as a function of tolerances. In these studies, the total cost includes inspection costs and the cost of deviation of performance due to variance of parameters from the target or design value.

It is still a common practice in the industry to assign tolerances by convention rather than by scientific considerations. In the development of the integrated decision system in this research, some of the Taguchi methods for modeling the quality control function will be applied. Further discussion of quality control, inspection, and process modeling will be presented in Chapter 5.

1.2.5 Optimizing the Manufacturing Cost

There have been numerous studies on the topic of optimum design for manufacture, optimum manufacturing plant design and scheduling problems. A few

examples of such studies of product design for optimum manufacturing cost are listed here. Hayes, Davis and Wysk [Haye81] presented an approach to determine the number of machines and their operating rates for each machine center in a production environment. They used a dynamic programming approach to solve the problem. Sutherland and Roth [Suth75] considered the manufacturing tolerances and costs in the design of mechanisms in function generating problems. The textbook by Trucks [Truc76] discusses the methods of economical production. Boothroyd articulated the need for developing and using economic criteria for design for economical production and product assembly. For example, he proposes [Boot84] methods for design of parts so that the assembly cost is minimized. He also presents [Boot85] the models for approximate cost estimating of turned parts. In his 1990 work [Boot90], he proposes models for costing procedures which are intended to form a basis of design for economical manufacture (DFM). Here he proposes the manufacturing cost models for machining and injection molding processes. The idea is to use these models to choose the appropriate materials and manufacturing processes at the design stage.

1.2.6 Investment in Manufacturing, Quality, and New Technologies

New accounting methods for estimating manufacturing costs are needed which will enable us to accurately model the cost of manufacturing including such intangibles as, flexibility, quality and new technology. This will also allow the designers to optimize the manufacturing costs at the design stage and make sound decisions on investments in manufacturing systems and new technologies. The present management

costing practices place too much emphasis on labor costs and do not properly measure intangibles such as production flexibility and product quality. Over the past few decades the cost of labor has been steadily decreasing as a percentage of the total cost of a product. Hence the practice of measuring overheads using labor cost is not justifiable anymore. Methods to measure the value of intangibles such as flexibility and quality have been proposed but are not yet practiced widely.

1.3 Traditional Product Realization Process

Typically, when developing an automated decision system for product design, the main objective is to optimize only the performance characteristic of the product. Once the optimum design parameters are found, the concerns of how to manufacture the product are addressed. If necessary, the design may involve one or more iterations in the design stage based on any new constrains found from the efforts to realize the design in manufacturing. These constraints typically result from the limitations of shop capacity, or capability, or possibly the lack of proper quality designed into the product. These iterations generally impose unexpected additional costs and delays of the product delivery. Later, as the product is used or operated, the customer's complaints and suggestions are fed back to the design engineer, thus requiring more design iterations in order to address the operation, quality, reliability and maintainability issues.

The above approach to product design has been the common method of product realization in most manufacturing industries. Subsequently there is little direct input

of manufacturing concerns, process capabilities, quality and maintainability for the design engineer to incorporate in the design decisions such that a universal optimum is obtained. Universal optimum in this context implies the minimum life-cycle cost with highest quality and reliability subject to given operational, manufacturing and technological constraints.

Figure 1.1 shows the stages in product life cycle. The dashed lines indicate the possible feedback presented for design iterations. Such iterations are a result of unimplementable or non-optimum design specifications such as shape or dimensions that are impossible to manufacture; tolerances that are unrealistically tight; materials that are costly, unsuitable to task or manufacture; performance characteristics that are unattainable as designed; or quality problems.

The decisions of normal product development/realization efforts at each stage of the product life cycle are sequential in nature. The output of one department (stage) is the input to the next department (stage). Often the decisions made in one department are constraints to the other department. Each department has different immediate concerns in realizing a product but ultimately has the same goal — build the best product at the lowest possible cost. Each department tries to minimize the cost of its own separate task, but one unit of cost reduction in one department may not necessarily lead to a decrease in overall costs. Thus individual departmental cost reduction decisions independent of other departments will in general lead to suboptimal life-cycle product design.

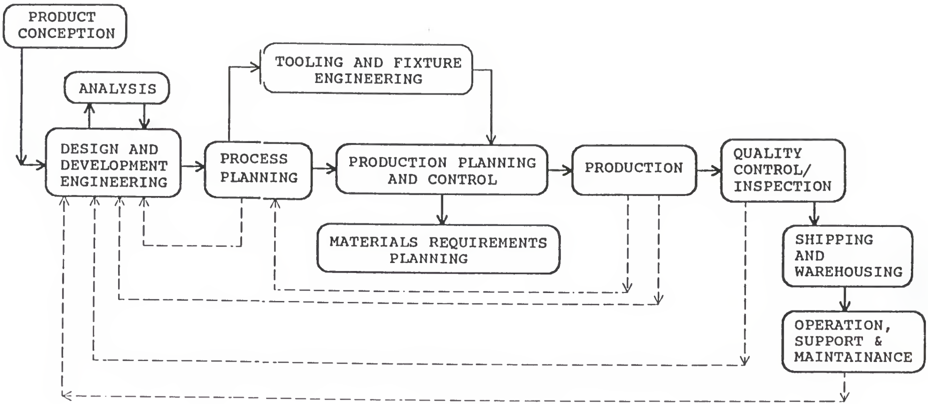


Figure 1.1: Stages in product life-cycle.

1.4 Modeling the Product Life Cycle

The life-cycle cost of a product can be defined as the cost incurred to society from the product conception stage to the product disposal stage. The stages in the product life cycle can roughly be describes as follows

- product planning and marketing
- research and development
- design and prototype
- manufacturing
- quality assurance
- operation, support and maintenance
- disposal

In this research, the overall life-cycle cost is considered to consist of

- manufacturing cost, C_M
- operation cost, C_O
- internal quality cost, C_{IQ}
- external quality cost, C_{EQ}

In order to design the products for optimum life cycle, costs in all the stages of the life-cycle as functions of various design, manufacturing, and quality control parameters can be formulated. Next, constrained optimization techniques can be used to optimize the life-cycle cost as a function of the parameters of design, manufacturing, and quality control.

The block diagram in Figure 1.2 represents the decision making process for the life cycle of a product as a set of serially connected stages. The following discussion assumes that there are *I* stages. The output of one stage is the input to the next stage of decision making and can have great influence on the subsequent stages. Often the

parameters of one stage act as constraints to another stage, thus restricting the design space. A stage of the life cycle can be represented by a block diagram as in Figure 1.3. The stage i has a set of input parameters (or state variables) $\boldsymbol{S}_{i-1}$, a set of decision variables (or stage design variables) $\boldsymbol{X}_i$, a set of transformation equations F_{ij}, an objective function U_i, and a set of output parameters $\boldsymbol{S}_i$.

The stage model for each stage is given by its transformation equations that map the stage input parameters and stage decision variables to the stage output parameters. The stage model receives input parameters, then in conjunction with stage decision variables and stage behavior, it produces the stage output. Thus, the output of stage i, $\boldsymbol{S}_i$, is a function of the input parameters $\boldsymbol{S}_{i-1}$ and stage decision variables $\boldsymbol{X}_i$. This functionality for stage i, called transformation equations, can be expressed as

$$\boldsymbol{S}_i = F_i(\boldsymbol{S}_{i-1}, \boldsymbol{X}_i), \text{ for } i = 1,\ldots,I.$$

The transformation equations, objective functions, and constraints could be of simple closed form, empirical charts, or they may require numerical modeling.

The objective function at each stage is the cost component associated with that stage. For example, the design stage objective function is the cost of R&D and product development and the operation stage objective function is the operation, support and maintenance for the life of the product. The stage objective function can be expressed as

$$U_i = U_i(\boldsymbol{S}_{i-1}, \boldsymbol{X}_i).$$

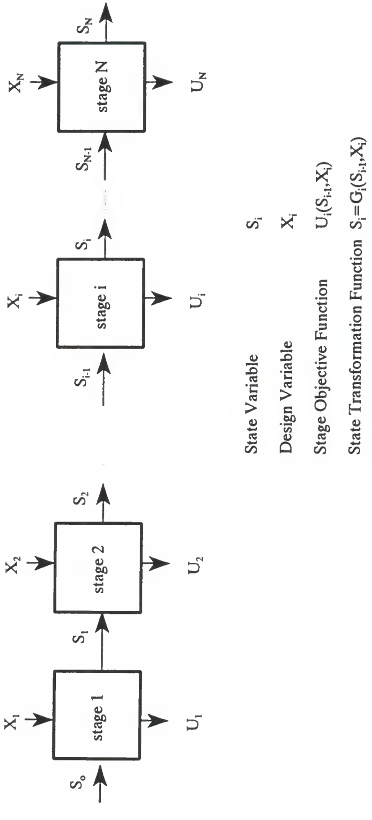


Figure 1.2: Product life-cycle as serially connected stages.

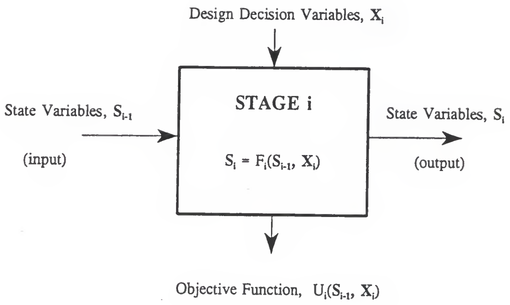


Figure 1.3: Stage i of the product life-cycle.

Optionally, a stage decision could be constrained by its given input and required output. These constraints for stage i can be expressed as

$$g_{ik} = g_{ik}(\boldsymbol{S}_{i-1}, \boldsymbol{X}_i, \boldsymbol{S}_i) \leq 0, \quad k = 1,\ldots,K_i$$

Finally, the overall objective function for all the stages combined is the life-cycle cost, which can be expressed as

$$U = U(\boldsymbol{S}_0, \boldsymbol{X}_1, \boldsymbol{X}_2,\ldots, \boldsymbol{X}_I) = \sum_i U_i(\boldsymbol{S}_{i-1}, \boldsymbol{X}_i)$$

To integrate the four functions of research and development, design, manufacturing, quality assurance, operations and maintenance into a single system, simplified models are developed in the following chapters to describe each stage.

1.5 Research Objectives

The objective of this research is to address the need for a comprehensive approach to the overall life-cycle cost optimization of products at the design stage. The focus of the present work is on developing an algorithmic framework for integrating the many separate efforts that deal with each stage of the product life cycle. A further objective of the research is to develop a methodology for performing the cost/benefit

analysis (sensitivity analysis) for making decisions to invest in plants, equipment, R&D and new technologies.

In this research, algorithmic and heuristic methodologies for life-cycle parameter tolerance design are proposed. The concept is based on optimizing a product's life cycle cost per hour of its operation. Life cycle cost consists of manufacturing cost, internal and external quality cost, and operating cost during the life of the product. These three components of the life cycle cost are strongly influenced by not only the parameters but also the tolerances for those parameters of the product design.

Manufacturing cost consists of material and machining costs. Quality cost is considered to consists of internal (in-plant) cost of reworking and scrapping, process deviation diagnosis cost and quality measurement cost. External quality cost (during usage by consumer) is the incremental operating cost due to variations in the manufactured quality. Quality control parameters such as control limits for quality control charts and frequency of quality measurement interval are obtained as a by-product of optimization process. Operating cost reflects the product performance and maintenance parameters during the life cycle of the product.

In order to determine the investments in manufacturing plant and equipment, research and development, or new technologies, trends in life-cycle cost as a function of the design parameters are explored. Research and development and new technology can generate new alternatives at different stages of the product life cycle and consequently expand the design space by shifting the design constraints. Such a shift

in design constraints may result in lower life-cycle costs caused by the benefits due to improved performance, reliability or quality of the product. Reasonable quantitative assumptions are made to estimate the cost of R&D and new technologies. But in a real-life situation, where reliable cost data are available, R&D cost can be determined more accurately. After design optimization and life-cycle cost determination, an exploration of the design space is done in order to determine the relative importance of design parameters and design constraints.

Hence a further objective of the research is to develop a methodology for sensitivity analysis of optimum design with respect to life-cycle decision parameters and active constraints. This is necessary for quantitative determination of the value added from research and development, design innovations, new technologies and new investments.

In this research, journal bearings with hydrodynamic and thermo-hydrodynamic models are taken as examples to illustrate the approach and the developed algorithms. The journal bearing life cycle is setup as a sequential optimization problem as depicted in Figure 1.2. To illustrate the developed methodology, parameter and tolerance design is undertaken for journal bearings. The clearance in a journal bearing strongly influences its performance and cost, and a careful tolerance design for clearance is critical for its optimum performance. The operating and waste disposal cost of the bearing is evaluated by simulating the energy loss and oil replacement during its life.

The research also deals with the development of an integrated knowledge based system for the design of journal bearings. Here heuristic suggested design is validated

using the algorithmic procedures. The class of problems considered here assumes a priori knowledge of the operating specifications, with the aim of generating an optimum bearing configuration.

1.6 Optimum Solution Strategies

1.6.1 Dynamic Programming Strategy

Though the problem is serial in nature, it is highly coupled mathematically, as we will see later. A special method which can be used to decompose (uncouple) the serial problem is the use of dynamic programming. Dynamic programming is particularly suitable for serial problems where the decisions taken at one stage influence only downstream stages. Because the optimization is carried out stage by stage, dynamic programming can be considered a decomposition technique. Following Bellman's principle of optimality [Bell57] each decision made at any stage takes into account the effects of that decision on downstream stages. The drawback of the dynamic programming is that it is very difficult to deal with the global constraints in the problem. However, it has the advantage of not requiring the continuity of the objective function. It is the basic approach used in this research for achieving the global optimum.

1.6.2 Multi-level Optimization

A general class of problems amenable to decomposition has a hierarchy of variables. The decomposition problem in such cases can be called multi-level optimization problems. This kind of a multi-stage (serial) problem can also be decomposed into a multi-level (hierarchical) problem. Multi-level is the second approach used in this research for achieving the global optimum and it is discussed in detail in Chapter 6.

1.6.3 Heuristic Based Strategy

The engineering design process is generally quasi-procedural, wherein heuristic suggested design is validated using algorithmic procedures. The class of problems considered here assumes a priori knowledge of the operating specifications, with the aim of generating an optimum bearing configuration. The task involves the preliminary synthesis of a suitable bearing configuration, dictated by a decision making capability and the user specified operating conditions. The preliminary configuration is analyzed to arrive at the optimum solution from a minimal manufacturing and life cycle operating costs standpoint. The procedure is iterative in nature, by which alternative configurations are synthesized using heuristics, and the best configuration is identified based on economic criteria. A novel feature of this approach is that the design constraint information is utilized as a feedback to the heuristic design segment of the program to improve the optimum design.

Most design decision systems, now available are usually limited to problem solving tasks in the deterministic or algorithmic domain. However, creative aspects of design are often ill-structured and require the judgement of the designer. Therefore attempts have been made in the past few years to incorporate qualitative decision making capabilities based on experience in design procedures [Dixo83, Liu87, Haje89]. In this research, an effort has been made to present a design methodology which integrates knowledge based methods and algorithmic analysis/optimization methods for optimum life-cycle design.

The following three chapters describe the four components of the life cycle cost. The journal bearing design problem is formulated in the next chapter. Operation and support stage is modeled in Chapter 3, where a bearing simulation procedure is developed in order to evaluate the bearing performance and calculate the cost of operation and support. The manufacturing and quality assurance stages and associated cost formulations are described in Chapter 5.

In Chapter 6, the optimization strategies are described in detail where the process of uncoupling the problem and making it into a truly sequential problem so that it can be formulated in terms of dynamic programming and multilevel optimization techniques is also discussed. This is followed by the incorporation of heuristic methods in the integrated design system. Chapter 7 provides a few examples of life-cycle optimization using the integrated design decision system. Chapter 8 presents a few examples in cost-benefit analysis of alternative investments in order to improve life-cycle cost. Finally,

Chapter 9 presents the summary of the work and concludes with some recommendations for further research.

CHAPTER 2
MODELING THE DESIGN STAGE

The objective at the design stage is to specify the detailed system parameters such that the desired system performance characteristics are achieved. After having designed a system, the design specifications can then be passed onto manufacturing, quality assurance and operations stages as input to these stages. The purpose here is not to model the design completely or perform an exhaustive analysis of all the minor variables. Rather, the purpose is to select the design parameters and performance characteristics that effect the manufacturing, quality, and operation and support costs. Another objective of the design stage is to investigate alternative designs; applicable alternative theories for design relationships; and perform cost/benefit analysis of investments in research, development and new technologies. This aspect of the design stage is elaborated in Chapter 6. In this chapter, journal bearing relationships for hydrodynamic and thermo-hydrodynamic lubrication, design specifications (stage inputs) and design variables (stage decisions) are described. The differences in analytical approach between hydrodynamic and thermo-hydrodynamic models of lubrication are discussed and algorithms for evaluating the performance are presented.

2.1 Journal Bearing Design Considerations

The nomenclature used to describe a journal bearing is depicted in a schematic diagram in Figure 2.1. The centers of the bearing and journal are at O_b and O_j respectively. The radial clearance *C* is the difference between the radii of the bearing and the journal. *L* is the length of the bearing. Figure 2.2 shows a schematic diagram for the design stage, illustrating the input parameters, decision variables and output variables. The main independent input parameters for the journal bearing under consideration are

- Bearing load (lb.), *W*
- Journal rotational speed (rps), *N*
- Journal diameter (in.), *D*

The main design decision variables to be determined by he designer are

- Bearing length to diameter ratio, *L/D*
- Radial clearance (in.), *C*
- Lubricant average viscosity (Reyn), μ
- Bearing material (strength, hardness, machinability, surface roughness and wear characteristics)
- Bearing configuration (shapes, holes, groves)
- Important tolerances — specifically tolerances on bearing and journal diameters (in.), Δ_D

The constraints on journal bearing design are considered as follows:

minimum oil film thickness,

$$h_o \geq h_{omin} = 5.0\times10^{-5} \text{ in}, \tag{2.1}$$

maximum outlet temperature,

$$T_o \leq T_{max} = 300 \text{ °F}, \tag{2.2}$$

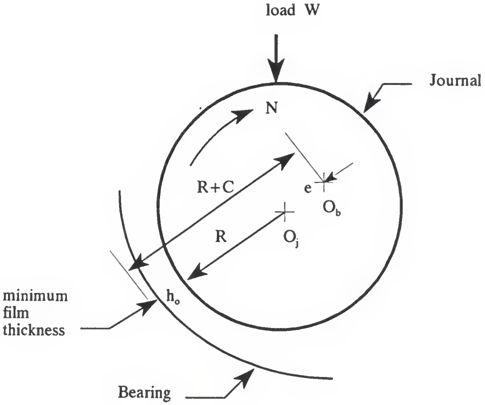


Figure 2.1: Journal bearing geometry and nomenclature.

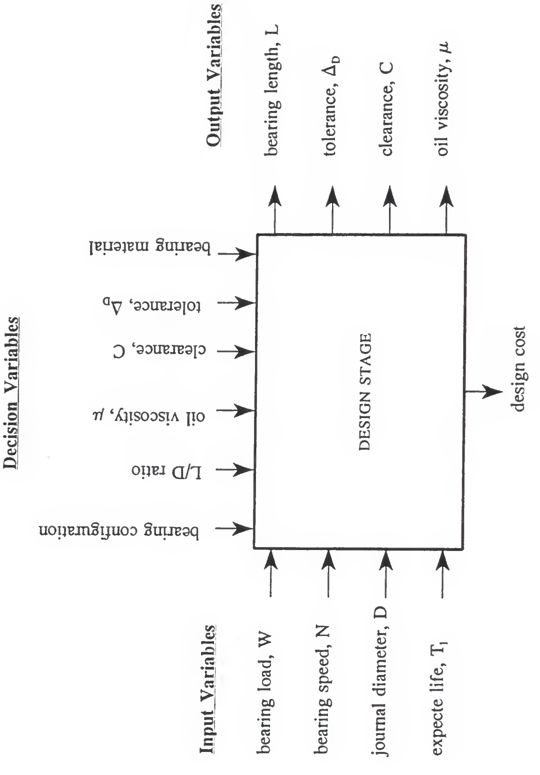


Figure 2.2: Schematic representation of design stage .

maximum pressure,

$$P_{max} \leq P_{max,u} = 30\times10^{3} \text{ psi}, \tag{2.3}$$

lower limit on the viscosity of the oil used,

$$\mu \geq \mu_{min} = 0.01\times10^{-6} \text{ Reyn}, \tag{2.4}$$

upper limit on the viscosity of the oil used,

$$\mu \leq \mu_{max} = 50.0\times10^{-6} \text{ Reyn}, \tag{2.5}$$

upper limit on length to diameter ratio,

$$L/D \leq (L/D)_{max} = 1.0, \tag{2.6}$$

lower limit on length to diameter ratio,

$$L/D \geq (L/D)_{min} = 0.25, \tag{2.7}$$

a criterion ensuring the dynamic stability of the bearing proposed by Lund and Saibel [Lund67], which relates the dimensionless rotor mass and a function of modified Sommerfeld number.

$$\frac{MC\omega^{2}}{W} \leq \psi(\sigma) \tag{2.8}$$

where

M	=	rotor mass per bearing (lb),
C	=	bearing radial clearance (in),
ω	=	journal rotational speed (rad/sec),
W_o	=	bearing load (lbf), and
σ	=	modified Sommerfeld number = $\pi S(L/D)^2$

$\psi(\sigma)$ is a function of ω, W_o, L, D, C and μ as presented by Lund and Saible [Lund67] and is illustrated in Figure 2.3. In order to compute ψ, the curve in Figure 2.3 was numerically modeled as

$$\sigma = \begin{cases} 3.0\sigma^{0.55}, & \sigma \leq 0.28 \\ 6.88\sigma^{0.094}, & 0.28 < \sigma \leq 2.9 \\ 7.65, & \sigma > 2.9 \end{cases} \tag{2.9}$$

2.2 Hydrodynamic Theory

In the analysis of journal bearing using hydrodynamic lubrication theory, the following physical quantities are assumed to be given.

- Bearing Speed N
- Load W
- Diameter D
- Clearance C
- Inlet oil temperature T_i
- Oil type (SAE grade)
- A function relating the oil viscosity μ to average temeperature T as follwos

$$\mu = F_\mu \mu_o \operatorname{Exp}[b_o/(T+95)] \tag{2.10}$$

Here μ_o is a constant representing the base viscosity for an oil 'o' — the SAE grade of the oil; b_o is a constant for oil 'o'; and F_μ is a deterioration factor representing the increase in viscosity due to contaminants and oil deterioration.

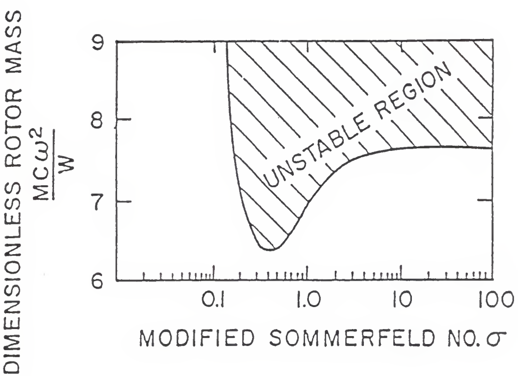


Figure 2.3: Journal bearing dynamic stability criterion developed by Lund and Saibel.

The design stage transformation relations are defined by the equations relating the bearing performance characteristics. In this section, the system of equations describing the hydrodynamic journal bearings design will be presented. The equations which govern the operating characteristics are the Reynolds differential equations for hydrodynamic journal bearing [Reyn86]. For the bearing design, the Raimondi and Boyd's numerical solution to Reynolds differential equations for a hydrodynamic journal bearing [Raim58] are among the most widely known. Their numerically solved results are given in the form of design charts, with Sommerfeld number (*S*) and length to diameter ratio (*L/D*) as the main parameters. These numerical solutions are based on the assumption that the film formed in the bearing is isoviscous and is independent of pressure and temperature variations. The film viscosity (μ) is calculated at the mean temperature in the bearing. Based on isoviscous hydrodynamic considerations, the effective Sommerfeld number S is defined as

$$S = f(\varepsilon, L/D) = \frac{\mu_a N}{P_a}\left[\frac{R}{C}\right]^2 \tag{2.10}$$

Here e is the journal eccentricity (distance between journal and bearing centers), ε (e/C) is the eccentricity ratio of the bearing. The objective of the journal bearing analysis is to find the following physical quantities in the bearing.

- Average temperature in the bearing T_{avg}
- Average pressure in oil film P
- Maximum pressure in oil film P_{max}
- Minimum oil film thickness h_o
- Coefficient of friction f
- Oil flow rate throgh the bearing Q

Specifically, the curve fittings by Seireg and Dandage [Seir82] to Raimondi and Boyd's design charts were used in this research. The curve fitted Seireg-Dandage relations describe various bearing parameters as a function of length to diameter ratio (*L/D*) and Sommerfeld number (*S*). The parameters used in this research which these equations describe are the temperature rise (ΔT), maximum pressure (P_{max}), oil flow rate (Q), coefficient of friction (f), and minimum film thickness (h_o) in the bearing as the following nondimensional quantities

$$\frac{h_o}{C},\ \frac{R}{C}f,\ \frac{P}{P_{\max}},\ \frac{Q}{RNCL},\ \frac{J\gamma c\Delta T}{P} \tag{2.11}$$

The equations for the above quanitities in [Seir82] are of the following form

$$\text{nondimensional quantity} = K_1 \left[\frac{L}{D}\right]^{K_2} S^{K_3+K_4\left(\frac{L}{D}\right)} \tag{2.12}$$

These equation are defined with different values of constants K_1, K_2, K_3, K_4 for different ranges of Sommerfeld number *S*, and length to diameter ratios *L/D*, for different nondimensional quantities.

One of the most important assumptions made in the Raimondi-Boyd numerical analysis is that the viscosity of the oil film is constant. As the work is done on the oil during flow through the bearing, the temperature of the oil rises. From the viscosity, temperature equation given above, the viscosity drops off with rising temperature. Since the analysis is based on constant viscosity, the task is to find the value of

effective temperature and viscosity to be used in the analysis. To do this, following two quantities are first defined

T_1 = Temperature evaluated using viscosity temperature relation given by equation (2.10).

T_2 = Temperature evaluated using temperature rise ΔT given by Seireg-Dandage curve fits to Raimondi and Boyd's solution.

Then T_1 and T_2 are iteratively changed in order to minimize the difference between the two temperatures ($|T_1$-$T_2|$). In order to minimize the temperature difference, golden sections directional search method is used. Now the effective oil film temperature is taken to be $T_{avg} = (T_1+T_2)/2$. Based on the T_{avg} just evaluated, h_o, f, Q, and P_{max} are evaluated using Seireg-Dandage curve fits given by relations of type in equation (2.11).

2.3 Thermo-hydrodynamic Theory

Thermo-hydrodynamic behavior model which takes into account the thermal effects in the journal bearing, and is more accurate in representing the temperatutres and pressures in the beaing, was also used for the design.

2.3.1 Difference From Hydrodynamic Theory

In the classical hydrodynamic theory presented by Reynolds, an isoviscous film is assumed. This simplification is widely used in bearing design since accounting for

the effects of temperature variations along the lubricant film and across its thickness are avoided. Many experimental observations, however, show that the isoviscous hydrodynamic theory, alone, does not account for the load carrying capacity and the temperature rise in the fluid film. A thermo-hydrodynamic phenomenon, as explained by Seireg and Ezzat [Seir73] and other researchers, occurs in fluid film bearings which influences the load carrying capacity of the film.

A procedure for journal bearing analysis based on thermo-hydrodynamic model, as proposed by Seireg and Dandage [Seir82], was also used to design the bearing. Seireg and Dandage found that only at certain oil inlet temperature, for any film geometry, oil and speed does the thermo-hydrodynamic phenomenon produce no effect on the pressure, and the hydrodynamic theory can be accurately applied. This condition is assumed to represent thermo-hydrodynamic equilibrium in the fluid film. The procedure for determining this equilibrium state (O^*) is as follows:

1. For a particular film geometry, speed and oil, a curve can be constructed for the average pressure P_a versus average film temperature T_a based on hydrodynamic theory as illustrated in Figure 2.4.
2. The point representing the thermo-hydrodynamic equilibrium state (O^*) is empirically found to be the point on this curve where the slope of the tangent is

$$\frac{dP_a}{dT_a} = \frac{K}{V} \tag{2.13}$$

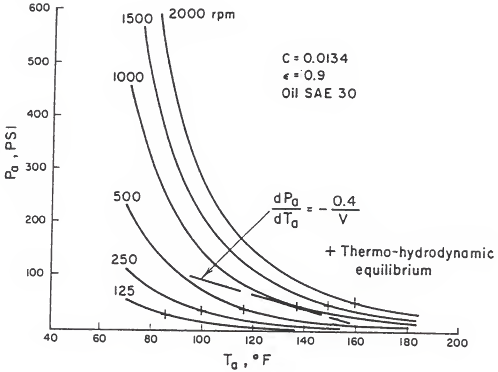


Figure 2.4: Determination of thermo-hydrodynamic equilibrium point O^* from pressure P_a versus temperature T_a curve for hydrodynamic lubrication of journal bearing [Seir73].

where

V = volume of oil drawn into the clearance space in cubic inches per revolution.

K = a constant which is found empirically to be a function of (R/C) as plotted in Figure 2.5 for all the oils and speeds.

3. The temperature rise ΔT^* at the O^* condition can readily be determined based on the isoviscous hydrodynamic theory. Consequently, the oil inlet temperature T_i corresponding to this condition can be calculated from

$$T_i = T_a^* - \frac{\Delta T^*}{2} \tag{2.14}$$

4. For the given eccentricity, oil grade, and inlet temperature T_i, the pressure speed relation can be empirically expressed as

$$\frac{(P_a)_{T_i}}{P_a^*} = \sqrt{\frac{N}{N^*}} \tag{2.15}$$

Since the pressure distribution as predicted by the isoviscous hydrodynamic theory remains the same, therefore,

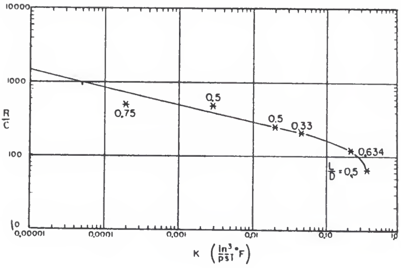


Figure 2.5: Determination of the thermo-hydrodynamic behaviour constant K from *R/C* ratio [Seir82].

$$\frac{(P_{\max})_{T_i}}{P^*_{\max}} = \sqrt{\frac{N}{N^*}} \tag{2.16}$$

This relationship is depicted in Figure 2.5 and compared to corresponding relationship predicted by isoviscous considerations for the same conditions.

2.3.2 Empirical Procedure for Analysis

For analysis of thermo-hydrodynamic behaviour of journal bearings, Seireg and Dandage defined a modified Sommerfeld number S^* [Seir82], and consequently an effective average viscosity. This accounts for the thermo-hydrodynamic behavior of the lubricant film and can be directly used instead of Sommerfeld number S to evaluate the performance characteristics for any operating conditions using existing isoviscous analysis and data. They defined S^* at equilibrium point O^* as

$$S^* = \frac{\mu_a^* N^*}{P_a^*}\left[\frac{R}{C}\right] \tag{2.17}$$

The Seireg and Dandage [Seir82] empirical procedure for determining the modified Sommerfeld number S^* is used. The S^* can be used instead of the classical Sommerfeld number (S) to determine the bearing performance characteristics using available data and methods based on the isoviscous hydrodynamic considerations. In this procedure the inlet temperature of the oil (T_i) in conjunction with temperature rise ΔT is used to obtain the modified Sommerfeld number by successive iterations. In this

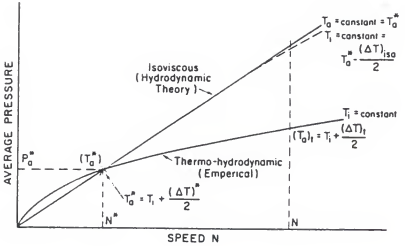


Figure 2.6: Pressure (P) and speed (N) relationship for a fixed geometry bearing [Seir82].

thermo-hydrpdynamic bearing analysis algorithm, the main objective is to find the modified Sommerfeld number S^*. The procedure is as follows.

(1). Find the bearing characteristic constant K from Figure 2.5 using the value of R/C. Then evaluate the parameter RCL/K for the bearing. For computational purpose, the curve in Figure 2.5 was modeled as a following relation

$$K = \begin{cases} 1.59312\times10^{9}(R/C)^{-4.50109}, & if\ (R/C)>200 \\ 10^{\alpha}, & if\ (R/C)<200 \\ \quad where\ \alpha = -7.0631+7.7637\log_{10}(R/C)-2.25814(\log_{10}(R/C))^2 & \end{cases} \tag{2.18}$$

(2). Calculate the value of the parameter

$$\frac{N}{P_a^2}\left(\frac{R}{C}\right)^2 \tag{2.19}$$

(3). Make an initial guess at T_a^* and P_a^*, the average temperature and pressure at the O* condition. Inlet temperature T_i and average pressure $P_a = W/LD$ are used as initial estimates.

(4). Compute the dimensionless oil factor

$$\frac{b\theta}{(T_a^*+\theta)^2} \tag{2.20}$$

corresponding to the current value of T_a^*.

(5). Compute the average viscosity corresponding to the current value of T_a^*.

$$\mu_a^* = Ae^{\frac{b}{T_a^*+\theta}} \tag{2.21}$$

(6). Calculate the approximation to S^* by using the formula

$$S^* = \mu_a^* P_a^* \frac{N}{P_a^2}\left[\frac{R}{C}\right]^2 \tag{2.22}$$

(7). If this approximation to S^* is sufficiently close to the previous approximation to S^*, there will no need for further iteration and approximation.

(8). Corresponding to the current value of S^* calculate the quantity of oil flow $Q/RNCL$ from Seireg-Dandage curve fitted equations.

(9). Estimate the new approximation to P_{a*} by using the equation

$$P_a^* = \frac{\left(\frac{Q}{RNCL}\right)\left(\frac{RCL}{K}\right)}{\left(\frac{b}{(T_a^*+\theta)^2}\right)} \tag{2.23}$$

(10). Corresponding to the current values of S^* and P_a^* find the temperature rise ΔT^* by using the curve fitted models of type given by equation (2.11).

(11). Revise the estimates for T_a^*

$$T_a^* = T_i + \frac{1}{2}\Delta T^* \tag{2.24}$$

(12). Go to step (4).

(13). Use the current value of S^* for further analysis of the bearing.

According to Seireg and Dandage, the bearing performance predicted using the above procedure for thermo-hydrodynamic model provides an excellent correlation with the experimental data published earlier in the literature.

CHAPTER 3
MODELING THE OPERATION AND SUPPORT STAGE

The aim of this chapter is to model the physical and economic life and the cost of operations and support during the life of the product. Specifically the example of journal bearing is used to develop the relationships between the operations and support cost, and the design and operating parameters. Only those costs which are influenced by the bearing design and manufacturing parameters are considered in cost modeling. The operating cost of the bearing is estimated to be the cost of the energy loss in the bearing during its life. The support cost is taken to be the cost of oil replacements in the bearing during its life which includes the disposal cost. In order to evaluate the total operation and support cost, bearing performance characteristics are simulated over its life as the bearing wears and its performance deteriorates. Empirical wear models dependent on operating conditions, and bearing and journal materials are used. The bearing life is taken to be minimum of its physical and economic life. The *physical life* is defined as the point in time when the bearing fails or one or more design constraint are violated and cannot be brought inside the feasible design domain even after replacing the oil. The *economic life* is defined as the point in time when the total-cost-rate curve has reached its minimum value and starts to rise with time. A schematic

diagram in terms of input, output and decision variables of the operation and support stage is shown in Figure 3.1.

3.1 Bearing Life Simulation

In order to evaluate the bearing life, and its operation cost, a simulation of bearing performance is done as the bearing wears over time. The underlying concept is that with time the bearing wears and the quality of oil deteriorates because of oxidation and presence of contaminants. The replacement of deteriorated oil with new oil brings the performance characteristics of the bearing closer to their designed optimum values.

3.1.1 Wear Modeling

The bearing clearance increases due to wear, hence the performance deteriorates from the optimum. It is understood that in reality the wear along the bearing surface (specially along the tangential direction) does not occur uniformly. Wear is more pronounced at the bottom compared to the top half of the bearing, leading to an elliptical shape over time. But because of the lack of a theory for modelling elliptical bearings, it is assumed in this research that the wear occurs uniformly along the bearing surface. Hence with time, the shape of both the bearing and the journal remain exactly circular.

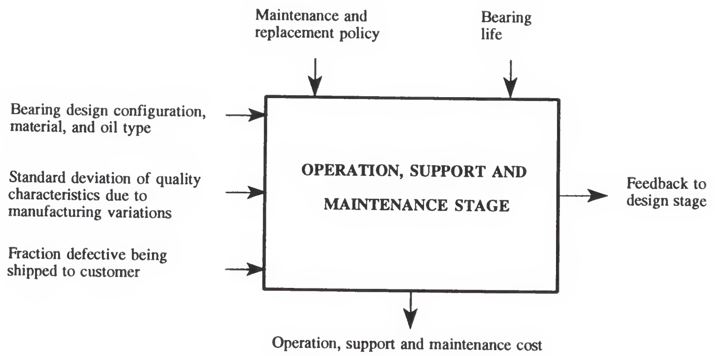


Figure 3.1: Schematic diagram of operation and support stage.

To predict the bearing wear rate, we propose a deterministic model which combines the empirical relationships for bearing wear rates with sound quantitative assumptions for statically loaded journal bearing. Many experimental studies have been conducted in the past to empirically model the journal bearing wear. One of the model proposed by Watanabe et al. [Wata85] expresses the bearing wear rate as a function of contaminant concentration in oil; contaminant particle size; hardness of contaminant, bearing and shaft material; and relative motion between shaft and bearing. Watanabe's model for rate of bearing wear volume per revolution can be expressed as follows,

$$\frac{dW_{vl}}{dn} \propto \left[K_b \mathrm{W}_{hb}(H_c,H_b,H_s) + K_s \mathrm{W}_{hs}(H_c,H_b,H_s)\right] r_{co}^{1.3}\, r_f^{-2}\, LD \tag{3.1}$$

Here W_{hb} and W_{hs} are functions of bearing, journal and shaft material hardness, and are empirically determined to be

$$\mathrm{W}_{hb}(H_c,H_b,H_s) = \begin{cases} \left(\dfrac{H_c}{H_b}\right)\left(\dfrac{H_c}{H_s}\right)^{0.25}, & \dfrac{H_c}{H_s} > 1 \\[2ex] \left(\dfrac{H_c}{H_b}\right)\left(\dfrac{H_c}{H_s}\right)^{15}, & \dfrac{H_c}{H_s} < 1 \end{cases} \tag{3.2}$$

Notations in the above three expressions are as follows

dW_{vl}/dn = combined wear rate of bearing and shaft by volume as modeled by Watanabe (in^3/revolution)

$$W_{hs}\,(H_c,H_b,H_s) = \begin{bmatrix} \left(\dfrac{H_c}{H_s}\right)\left(\dfrac{H_c}{H_b}\right)^{0.5}, & \dfrac{H_c}{H_s} > 1 \\ \left(\dfrac{H_c}{H_s}\right)^{8}\left(\dfrac{H_c}{H_b}\right)^{0.5}, & \dfrac{H_c}{H_s} < 1 \end{bmatrix} \tag{3.3}$$

K_b, K_s = wear coefficients for bearing and shaft material respectively

L = bearing length, (in)

D = bearing diameter, (in)

r_{co} = ratio of contaminants volume to total oil volume (including the oil in sump and pump),

H_c, H_b, H_s = hardness of contaminant particles, bearing and shaft material respectively,

N = bearing speed, (rps)

r_f = ratio of minimum film thickness to average diameter of contaminants particles

$$r_f = \frac{h_o}{d_p} \approx \frac{h_o}{h_{o,\min}} \tag{3.4}$$

Note that in the above equation, it has been assumed that the average contaminant particle size (d_p) in the oil is approximately equal to the lower limit on minimum oil film thickness ($h_{o,\min}$) in the bearing.

Watanabe et al. did not take into account the effects of oil temperature change, pressure, and coefficient of friction [Wata85]. In order to take the above effects into account, the following model for the rate of wear volume is proposed

$$\frac{dW_{v2}}{dn} \propto (f\, P_{max}\, T_o)^{1/2} \tag{3.5}$$

where

f	=	coefficient of friction for the oil film,
P_{max}	=	maximum oil pressure in the film (psi),
T_o	=	bearing outlet oil temperature (°F).

Hence the total wear volume ΔW_v, resulting in Δn bearing revolutions after taking into account the effect of material properties in equation 3.1 and bearing performance characteristics in equation 3.5, can be expressed as follows

$$\Delta W_v = \frac{dW_{v1}}{dn}\,\frac{dW_{v2}}{dn}\,\Delta n \tag{3.6}$$

Here Δn, the number of bearing revolutions in Δt hours, is

$$\Delta n = 3600\, N \Delta t \tag{3.7}$$

Finally the average increase in bearing clearance ΔC (in) due to wear in time Δt (seconds) can be derived as

$$\Delta C = \frac{dW_{v1}}{dn} \frac{dW_{v2}}{dn} \frac{3600N\Delta t}{\pi DL} \tag{3.8}$$

3.1.2 Modeling Oil Deterioration

Viscosity of oil is assumed to change with time because of deterioration of oil quality and generation of metallic particles due to wear and the presence of contaminant in the oil. The change in viscosity of oil is reflected by addition of a coefficient F_μ in viscosity-temperature relationship of the oil given by equation 2.10 and is repeated here

$$\mu = F_\mu \, \mu_o \, \mathrm{Exp}[b_o/(T+95)] \tag{3.9}$$

where

μ_o = A constant representing the base viscosity for an oil 'o',

b_o = A constant for oil 'o',

F_μ = A 'deterioration factor' representing the change in oil viscosity due to contaminants and deterioration.

For a newly replaced oil, F_μ is unity and it varies with the number of bearing revolutions, modeled as follows

$$F_\mu = F_{\mu i} \, (1 + n/n_o)^{1/2} \tag{3.10}$$

where

$F_{\mu i}$ = 1,

n = number of revolution in the bearing since last oil replacement,

n_o = a constant, assumed to be 2×10^6 revolutions.

After combining the empirical model suggested by Watanabe et al. with the effects of friction, temperature and pressure, we get the increase in clearance ΔC in Δn number of revolutions (or, in time interval $\Delta t = \Delta n/3600N$ hours) as follows

$$WF = \left[K_b\, f_1(H_c,H_b,H_s) + K f_2(H_c,H_b,H_s)\right] r_{co}^{1.3} \left[\frac{h_o}{h_{omin}}\right]^2 \tag{3.11a}$$

$$\Delta C = WF\ (fP_{max}\ T_o)\ 3600N\Delta t \tag{3.11b}$$

After many simulations, the following expression was developed for contaminats to oil ratio

$$r_{co} = r_{co,i} + \frac{\pi LD}{V_o}(C-C_{no})\left[1+\frac{t_{no}}{t_o}\right] \tag{3.11c}$$

where,

$r_{co,i}$ = initial ratio of contaminats to oil (immediately after the old oil is replaced),

L,D = bearing lenght and diameter,

$C =$ current bearing clearance,

$C_{no} =$ bearing clearance at last oil replacement (new oil),

$V_o =$ total oil volume including the oil in sump and pump,

$t_{no} =$ hours of operation since last oil replacement,

$t_o =$ a constant (assumed to be 50.0).

3.1.3 Modeling Bearing Instability

As the bearing wears and the clearance increases, the bearing may tend towards dynamic instability. As the dynamic instability is approached in the bearing as modeled by equation 2.8, the dimensionless rotor mass ($MC\omega^2/W$) approaches $\psi(\sigma)$. The approach towards dynamic instability is not a sudden phenomenon, but rather an incremental phenomenon. The approach towards instabilities causes fluctuating loads on the bearing, resulting in increased effective load. In order to account for this fact, a factor called *dynamic load multiplier* (λ), which is modeled as follows

$$\lambda = \frac{1}{1 - \left[\dfrac{MC\omega^2/W_o}{\psi(\sigma)}\right]^3} \tag{3.12}$$

is multiplied to the *specified static load* W_o. The dynamic load multiplier increases the *effective dynamic load* (W) on the bearing as follows

$$W = \lambda W_o. \tag{3.13}$$

The dynamic load on the bearing tends towards infinity as the dimensionless rotor mass ($MC\omega^2/W_o$) approaches the Lund and Saibel instability criterion $\psi(\sigma)$ [Lund67]. In all the calculations for evaluating bearing performance, the effective bearing load (W) is used instead of the specified static load W_o. Only in evaluating the bearing stability criterion, used for determination of bearing life, is the specified static load W_o used.

3.1.4 Operation Cost Model

Operation cost $C_o(t)$ (a function of operation time) of the bearing consists of the cost of energy loss due to friction $C_{oe}(t)$ and the cost of lubricant replacement $C_{or}(t)$ over the life of the bearing, and is expressed as

$$C_o(t) = C_{oe}(t) + C_{or}(t) = r_e H(t) + r_o V_o$$

or

$$C_o(t) = \sum (r_e bJQ\rho c_s \Delta T + r_o V_o)\Delta t \tag{3.14}$$

where

t	=	bearing operation time,
Δt	=	operation time step (hrs) used during simulation,
r_e	=	cost rate of energy (0.01 \$/kWhr),
$H(t)$	=	energy loss in bearing due to friction (kWhr),
r_o	=	cost rate for oil (0.04 \$/in^3),
V_o	=	total volume of the oil to be replaced (in^3),

b = energy conversion factor, (1.054 Btu/kWhr),

J = mechanical equivalent of heat (9336 lbf-in/Btu),

ρ = oil density (0.0311 lbf/in^3),

c_s = heat capacity of oil (0.42 Btu/lbf-°F),

Q = oil flow rate (in^3/sec),

ΔT = temperature difference between oil inlet and outlet (°F),

Total operation cost of the bearing is evaluated by simulating the bearing performance over its life and using the summation in equation 3.14 over time.

3.2 Simulation Procedure

A block diagram explaining the bearing simulation is given in Figure 3.2. The bearing simulation is performed until the end of its useful physical or economic life, whichever occurs first. During bearing simulation, as the clearance increases, one or more design constraints are violated, causing oil to be replaced. The bearing is assumed to have reached the end of its physical life when replacing the bearing oil does not bring the bearing back into its feasible (valid) design domain. The bearing *economic life* is defined as the time of operation when the cost-rate (life cycle cost per hour of operation) has reached its minimum. The bearing is simulated using the following procedure.

(1). The following quantities are assumed to be given (input to subroutine):

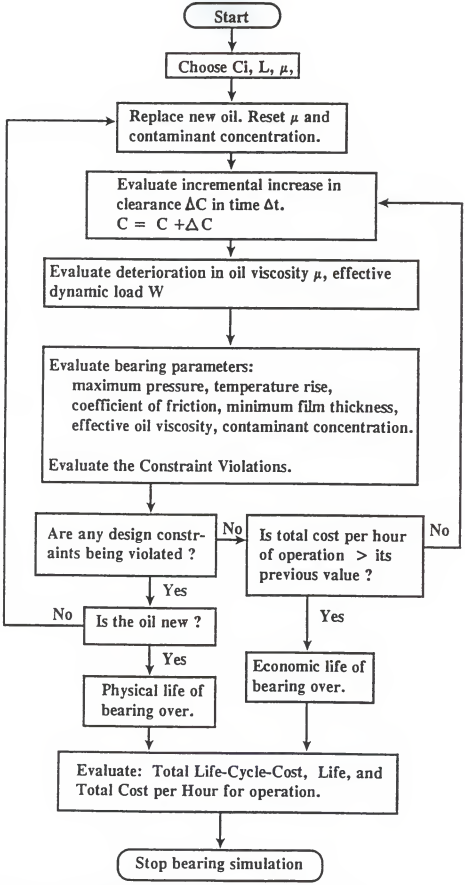


Figure 3.2: Bearing performance simulation over its life.

diameter D; L/D ratio; initial clearance C_i; viscosity evaluation coefficient G_μ.

(2). Initialize the following variables:

simulation time t=10; life=0; augmented Lagrange function, ALF=0; dynamic load W=W_0; cost-rate C_r=0; energy loss cost C_e=0; oil cost C_o=0; life-cycle cost LCC=0;

(3). Calculate the viscosity μ at inlet temperature T_i using equation 3.9.

(4). Initialize the following parameters:

time to replace oil t_{or}=0; number of oil replacements n_{or}=0; number of revolutions n=0; constraint number violated v=0; viscosity evaluation coefficient F_μ=G_μ;

(5). Using the hydrodynamic or thermo-hydrodynamic analysis procedure discussed in Chapter 2, evaluate the bearing performance. The following parameters are:

oil type 'o'; bearing speed N; specified static load W_o; bearing diameter D; bearing length L; oil inlet temperature T_i; viscosity coefficient F_μ.

The golden section search procedure is used to evaluate effective oil temperature T_{avg} and oil viscosity μ. Using the effective temperature T_{avg} and viscosity μ, evaluate the Sommerfeld number S and then using Seireg-Dandage equations, evaluate:

coefficient of friction f; oil flow rate Q; maximum pressure P_{max}; and outlet oil temperature T_o.

(6). Evaluate dynamic load multiplier λ using equation 3.12 and dynamic load W using the equation 3.13.

(7). Evaluate whether the following bearing performance constraints are being satisfied:

C1: $h_o > h_{o,min}$

C2: $P_{max} < P_{max,u}$

C3: $T_o < T_{max}$

C4: $\mu > \mu_{min}$

C5: $\mu < \mu_{max}$

C6: $r_{co} < r_{co,max}$

C7: $(L/D) > (L/D)_{min}$

C8: $(L/D) < (L/D)_{max}$

C9: $C > C_{min}+\Delta_D$

C10: $C < C_{max}-\Delta_D$

C11: Lund and Saibel stability criterion of equation 2.8 and Figure 2.3. Note that here, static load W_o is used to evaluate the dimensionless rotor mass, rather than dynamic load W.

C12: $t_{life} > t_{life,min}$

C13: $\Delta_D > \Delta_{D,min}$

If any constraint is being violated, initialize the variable V_i with that constraint number and append it to a string $V_{i,str}$ which stores the history of constraint violations during the life simulation. The constraint for the life of the bearing

(C12) is evaluated only at the end of life, when the constraint values are also calculated and stored in the constraint vector (**G**) after multiplying by a constraint satisfaction factor (*CSF*) as described in the overall optimization procedures given in Chapter 6.

(8). Calculate the cost of oil replacement $C_{o,o}$ and cost of energy loss $C_{o,e}$.

CHAPTER 4
MANUFACTURING AND QUALITY ASSURANCE STAGE

In this chapter, cost models for manufacturing and quality are developed. Manufacturing cost consists of material and machining costs. Quality cost is considered to consist of internal (in-plant) cost of reworking and scrapping, process deviation diagnosis cost and quality measurement cost. External quality cost (during usage by consumer) is taken to be the incremental operating cost due to variations in the manufactured quality. Optimum values of quality control parameters such as control limits for quality control charts and frequency of quality measurement interval are also obtained.

4.1 Manufacturing Cost Model

After the design stage, the manufacturing engineer develops a process plan which recommends the machine to be used and the operating parameter to use for each machine, such as speed, feed, depth of cut, etc. The machine parameters chosen and tooling used are usually based on an optimization algorithm to either utilize the maximum production rate of the machine or minimize the cost to process the part. The decisions of machine selections are based on the ability of the machine to create the

required geometry, capacity of the machine (power, speed etc.), and the capability to meet specifications, in conjunction with the results of the machine parameter optimization. Manufacturing cost is heavily dependent on the design specifications, and therefore on the machines selected and the parameters used by the process planner. A schematic diagram in Figure 4.1 shows the modeling of manufacturing stage in terms of its input, output and decision variables.

For the journal bearing example, the main concern for the designer will be specifying tolerance for diameter, bulk and surface material properties of the journal bearing such as strength, hardness, and surface roughness. Final processing steps such as fine turning, grinding, lapping and surface treatment such as coating will depend on designer's specifications. The following factors must be considered by the process engineer for selecting the process and machines:

Capacity — It is the size of parts the machine can handle such as diameter, length, power and speed of the machine.

Capability — It is the capability of the machines and processes to attain specifications such as geometry, dimensional tolerances, and surface roughness. Such capability for a quality characteristic of interest x can be expressed by a parameter such as process capability index

$$C_{p,x} = \frac{USL_x - LSL_x}{6\sigma_x} \tag{3.13}$$

where

USL_x = upper limit of design specification for x,

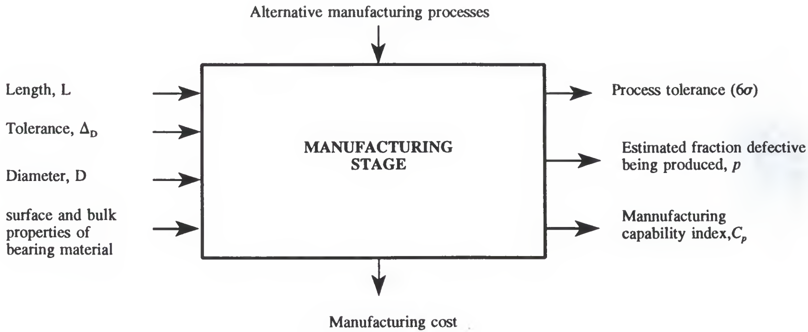


Figure 4.1: Schematic diagram of the manufacturing stage.

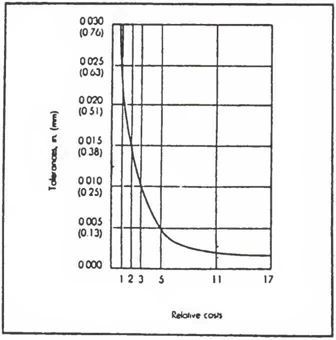
LSL_x = lower limit of design specification for x,

σ_x = standard deviation of the process for x.

Availability — It is a factor varying between zero and one, which shows whether the machine is available for use when requested, or difficult to obtain. This factor can be used to apply a cost penalty for choosing to use a relatively unavailable machine.

Cost — The cost to process an order on a given machine is a function of the lot size, setup time, fixture and tooling requirements, material handling time, processing time, and the base cost rate (cost per unit time for the machine). Also the cost depends on the design specifications such as tolerances, surface finish, and coatings etc.

Very little data is available in the literature for the cost of processing as a function of all the variables mentioned in this section. This kind of cost information may be internally generated within the manufacturing department from the machine history databases. The cost factors thus derived can be very useful to the design and process engineers in optimum design and machine/process selection. A few manufacturing handbooks which present the general trends in manufacturing cost as a function of tolerance and surface roughness [ASLE, Neal73, Truck76, Tool83] were consulted. Some of these relationships are shown in Figure 4.2 and Figure 4.3. Generally the following functions, which relate the manufacturing cost to tolerance or surface roughness, can be found in these handbooks



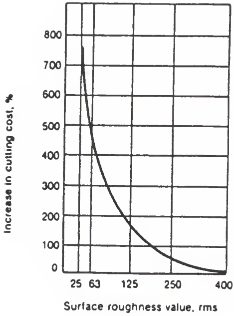


Figure 4.2: Manufacturing cost as a function of tolerance and surface roughness.

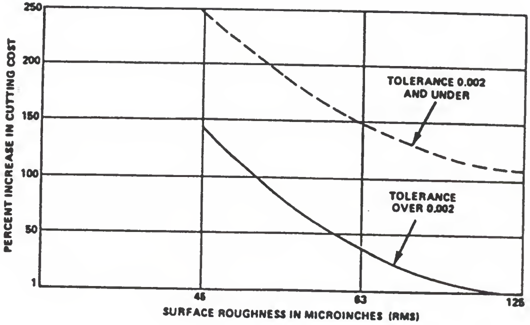


Figure 4.3: Manufacturing cost as a function of tolerances and surface roughness found in handbooks.

$$C_m \propto A + B/y \tag{4.2a}$$

$$C_m \propto A + B/y^2 \tag{4.2b}$$

$$C_m \propto A + Be^{-Cy} \tag{4.2c}$$

where y may be tolerance or surface roughness. Seireg and Corser [Cors85] modeled the base manufacturing cost for a journal bearing as follows

$$C_m = (A\Delta_D s + B\Delta_D + Cs + D)(39L + 12) \tag{4.3}$$

where A, B, C, D are constants; Δ_D is tolerance on bearing diameter; s is surface roughness; and L is the bearing length. In this thesis, manufacturing cost of the bearing depends on the bearing configuration, material, length (L), diameter (D), and tolerance on diameter (Δ_D) and is modeled as,

$$C_m = \alpha\beta\gamma\phi C_{m,base} \tag{4.4}$$

and

$$C_{m,base} = k_2 + k_3 L D^2 \tag{4.5}$$

where

α	=	bearing configuration cost factor, with the simple journal bearing having a value 1,
β	=	material cost factor, with the lead based babbitt having a value 1,
γ	=	machining cost factor based on the material hardness,
ϕ	=	tolerance cost factor = $(k_o + k_1/\Delta_D^{\alpha})$,
k_o-k_3	=	cost coefficients: 0.2, 0.5, 25, 1 respectively.

$a \quad = \quad 1/3.$

For the purpose of this research, we assume that bearing has a simple configuration, and is made of lead based babbitt, hence α, β, and γ are taken as 1.

4.2 Quality Cost Model

Quality costs are the costs incurred because poor quality may exist or it does exist. Taguchi defines the quality costs [Tagu78a] as the "loss imparted to the society by the product from the time the product is shipped." Generally quality costs are categorized as follows.

Prevention costs: These are costs incurred to prevent poor quality from being produced. Example of prevention activities include analysis and planning for quality, reliability, quality training, and development of process controls. The fixed part of these costs are not needed, since the fixed cost do not change in relation to the product parameters to be designed.

Appraisal costs: These are costs that result from the activities undertaken to prevent poor quality products from being further processed beyond the point at which they become non-conforming or from being delivered to customers. Example of appraisal activities include the inspection and testing of raw materials and products being produced.

Internal failure costs: These are costs associated with materials and products that fail to meet quality standards and are identified before the product or service is

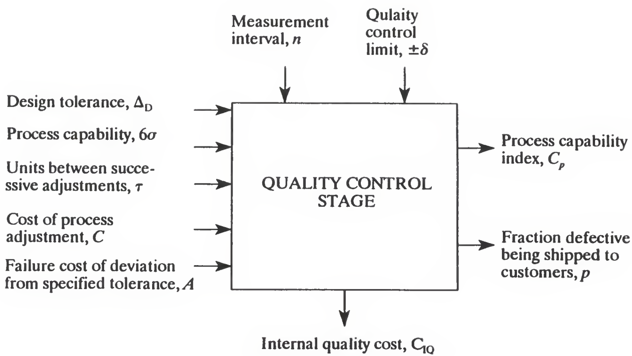


Figure 4.4: Schematic diagram of the quality control stage.

delivered to the customer. Examples of this include the scrap and spoilage, repair, rework, reinspection and downtime because of the quality problems.

External failure costs: These are the costs incurred because poor quality products are delivered to the customers. Examples include the costs of handling complaints, warranty replacements, product liability, and loss in performance of the product over its life.

In the following sections, the first three cost components will be referred to as *internal quality costs* and the last component is referred to as *external quality costs*. A schematic diagram in Figure 4.4 illustrates the quality control stage in terms of its input, output, and decision variables.

4.3 Internal Quality Cost Model

The cost of internal quality is measured using the concept of Taguchi's *on-line feedback quality control*, wherein measurements of product quality characteristics are obtained and analyzed, and the results fed back to upstream processes for their adjustments. Such measurements and adjustments of the process can be made either automatically or by a human operator. During the production, it may not be possible to measure every product, hence an attempt is made to measure and inspect a fraction of the products being produced. The major assumption made in deriving the quality costs in the following sections is that only discrete products are under production and the quality characteristic of importance is a continuous variable (e.g. diameter or length)

as opposed to an attribute (e.g. go/no-go, pass/fail). For the journal bearing problem, the quality characteristic of interest is the diameter of the bearing and the shaft. *Prevention and appraisal cost* have been assumed to consist of the following two components:

1. Cost of diagnosing (inspecting) the quality characteristic per unit of the product produced,

$$C_{IQ1} = B/n \tag{4.6}$$

where

B = cost per measurement of the quality characteristic,

n = measurement interval, i.e. measurements made at every n product.

2. Cost of process adjustment per unit of product produced,

$$C_{IQ2} = A/u \tag{4.7}$$

where

A = cost per process adjustment,

u = predicted average number of products between successive adjustment in the process.

Internal failure costs are actually the internal (in-plant) losses caused by deviations in a products quality characteristic from its target value. Taguchi proposed the concept of *loss function* for the quality characteristic of interest, which is the "loss

a product causes to society (in-plant loss and loss to customers) due to deviation in product quality characteristic from its nominal specified value." In particular, a quadratic loss function for internal failure costs is proposed, given by

$$C_{IQ3} = k\sigma_o^2 \tag{4.8}$$

where

σ_o^2	=	standard deviation of the quality characteristic of interest for the manufacturing process,
k	=	a constant.

If the *design tolerance* of the quality characteristic (diameter) is Δ_D and a deviation in diameter D outside its tolerance limits $D_o \pm \Delta_D$ causes an internal failure cost (due to rejection) A, then,

$$A = k\Delta_D^2 \tag{4.9}$$

or

$$k = A/\Delta_D^2. \tag{4.10}$$

Hence, internal failure costs due to a standard deviation σ_D in quality characteristic can be given by

$$C_{iq3} = \frac{A}{\Delta_D^2}\,\sigma_D^2 \tag{4.11}$$

where

A = in-plant cost of rework or scrapping a unit that falls outside the design tolerance interval $D_o \pm \Delta_D$, (\$)

D_o = target or nominal value of bearing diameter, (in)

Δ_D = design tolerance for diameter.

Let the control limits for the production process adjustment be set at $D_o \pm \delta$. When the production process is found to be in statistical control, the variance of D is given by

$$\sigma^2 = \delta^2/9 \tag{4.12a}$$

if the manufacturing process follows a normal distribution, and

$$\sigma^2 = \delta^2/3 \tag{4.12b}$$

if the manufacturing process follows a uniform distribution. When the production process is out of statistical control, the variance is given by,

$$\sigma^2 = \left[\frac{(n+1)}{2} + \tau\right] \frac{\delta_D^2}{u} \tag{4.13}$$

where

n = process measurement interval in units of products,

$\tau =$ lag in the process, i.e. number of units produced from the time the an out-of-control process is detected to the time the production process is stopped for recovery,

$u =$ predicted average number of products between successive adjustments in the process.

Usually u is not known in advance, hence, in order to evaluate the current value of u, a few trials are needed. If at a time when the quality control limits are set at $\pm\delta_o$, the value of u is u_o, then u at quality control limits δ is modeled as,

$$u = \frac{u_o}{\delta_o^2}\delta^2 \tag{4.14}$$

The above expression is derived from the assumption that the average time for a randomly-moving process to deviate by a certain amount is proportional to the deviation squared.

The total internal quality cost per product (C_{IQ}) is now modeled as the sum of the above three costs — cost of diagnosis (C_{IQ1}), cost of process adjustment (C_{IQ2}), and internal failure cost (C_{IQ3}),

$$C_{iq} = \frac{B}{n} + \frac{C}{u} + \frac{A\delta^2}{\Delta_D^2}\left[\frac{1}{3} + \frac{1}{u}\left(\frac{n+1}{2}+\tau\right)\right] \tag{4.15}$$

4.4 External Quality Cost Model

External quality cost is assumed to consist of only one of the components of the quality costs described in section 4.2. It is the component of operation and maintenance cost caused by the variations (deviation) in manufactured quality from the design specifications. Variations in many design parameters of the product from their optimum values cause the product to perform suboptimally. In the case of the journal bearing it may be the clearance (i.e. the bearing and journal diameters); the surface roughness and other parameters may also cause external quality costs to be affected. Here only the variations in clearance are assumed to cause external quality costs. We know that operation cost (C_o) of the bearing is a function of the initial clearance C_i (i.e. for a new bearing),

$$C_o = f(C_i). \tag{4.16}$$

Let the actual initial clearance for the new bearing vary in the range $C_{im} \pm \Delta_C$ with standard deviation σ_C due to manufacturing variations. Hence the expected value of the operation cost can be approximately given by

$$E[C_o] \approx f(C_{im}) + \frac{1}{2} \left. \frac{\partial^2 f(C_i)}{\partial C_i^2} \right|_{C_i = C_{im}} \sigma_{C_i}^2 \tag{4.17}$$

Since it can generally be assumed that C_i is a random variable having normal distribution, its variance in terms of manufacturing tolerance can be given by

$$\sigma_C^2 = \Delta_C^2/9.$$

Hence the expected value of the operation cost can be expressed as

$$E(C_o) = C_o(C_i^*) + \frac{\sigma_{C_i}^2}{2} \left. \frac{\partial^2 C_o(C_i)}{\partial C_i^2} \right|_{C_i=C_i^*} \tag{4.18}$$

where σ_{Ci}^2 is the variance of initial clearance from the manufacturing process. The clearance in the bearing is the difference between the bearing and journal radii

$$C = D_b/2 - D_j/2, \tag{4.19}$$

hence we have

$$\sigma_C^2 = \sigma_{Db}^2/2 + \sigma_{Dj}^2/2. \tag{4.20}$$

Assuming that

$$\sigma_{Db}^2 = \sigma_{Dj}^2 = \sigma_D^2 \tag{4.21}$$

and that the diameter D is normally distributed, in the worst case we can assume that $2\Delta_D=6\sigma_D$, i.e.

$$\sigma_C^2 = \sigma_D^2 = \Delta_D^2/9. \tag{4.22}$$

Finally the external quality cost from variations in bearing clearance due to manufacturing deviations can be expressed as

$$E(C_o) = \frac{\Delta_D^2}{18} \left. \frac{\partial^2 C_o(C_i)}{\partial C_i^2} \right|_{C_i = C_i^*} \tag{4.23}$$

Note that the operation cost curve around optimum initial clearance C_i^* must be known before external quality cost could be calculated.

4.5 Optimal Quality Level

Now that the internal and external quality costs have been modeled, we can determine the optimum quality control parameters by setting the derivative of C_{IQ} with respect to these parameters as zero. This simple process gives the following optimum values for the quality control parameters. The optimal diagnosis interval for the production process obtained by substituting the expression for u from equation 4.14 into equation 4.15 and setting its derivative with respect to n to zero. This results in optimum process measurement interval n^* as follows

$$n^* = \frac{\Delta_D}{\delta_o} \left[\frac{2 u_o B}{A} \right]^{1/2} \tag{4.24}$$

Optimum quality control limit during the production is derived by differentiating the total quality cost function in equation 4.15 with respect to δ and setting it to zero. This results in optimum quality control limits ($D_o \pm \delta^*$) as follows

$$\delta^* = \left[\frac{3C\delta_o \Delta_D^2}{Au_o} \right]^{1/4} \tag{4.25}$$

The predicted optimum manufacturing process capability index C_p, defined as the ratio of design tolerance (2Δ) and six sigma (6σ) of the manufacturing process is given by

$$C_p = \frac{2\Delta_D}{6\left[\frac{\delta^{*2}}{3} + \left(\frac{n^*+1}{2} + \tau \right) \frac{\delta^{*2}}{u} \right]^{1/2}} \tag{4.26}$$

Substituting the expressions for u^* in equation 4.24 and δ^* in equation 4.25 into equation 4.15 gives the following expression for optimum internal quality cost

$$C_{iq}^* = \frac{K_{iq1}}{\Delta_D} + \frac{K_{iq2}}{\Delta_D^2}, \tag{4.27}$$

where the two constants are as follows

$$K_{iq1} = \left(\frac{1}{\sqrt{\tau}} + \frac{1}{2} \right) \left(\frac{AB}{u_o} \right)^{1/2} \delta_o + 2 \left(\frac{CA}{3u_o} \right)^{1/2} \delta_o \tag{4.28}$$

and

$$K_{iq2} = \frac{A}{u_o} \left(\frac{1}{2} + \tau \right) \delta_o^2 \tag{4.29}$$

The methods of quality control presented in this chapter are referred to as *on-line quality control*. Unlike the traditional practice of the quality control, here the concept of feedback in a quality control system is used. We determine the optimal control

limits for the quality characteristic of interest, checking intervals, and process capability index that minimize the total cost of production.

CHAPTER 5
THE DYNAMIC PROGRAMMING APPROACH

The decisions of normal product development/realization efforts at each stage of the product life-cycle are sequential in nature. The output of one department (stage) is the input to next department (stage). Often the decisions made in one department are constraints to the other department. Each department has different immediate concerns in realizing a design, but ultimately have the same goal — build the best product at the lowest possible cost. Each department tries to minimize the cost of its own separate task, but one unit of cost reduction in one department may not necessarily lead to decrease in overall costs. Thus individual departmental cost reduction decisions independent of other departments will in general lead to suboptimal life-cycle product design.

5.1 Dynamic Programming Problem Formulation

The serial nature of the problem, lends itself to the use of dynamic programming solution strategy. Dynamic programming is particularly suitable where the decisions taken at one stage influence only 'downstream' stages. The dynamic programming method is an efficient computational solution strategy for this kind of serial problems.

Because the optimization is carried out stage by stage, dynamic programming can be considered a decomposition technique. Following Bellman's principle of optimality [Bell57] each decision made at any stage takes into account the effects of that decision on downstream stages.

The drawback of the dynamic programming is that it is very difficult to deal with the global constraints in the problem. However, it has the advantage of not requiring the continuity of the objective function. Dynamic programming is the basic approach used in this research for achieving the global optimum.

The block diagram in Figure 1.2 represents the decision making process for the life cycle of a product as a set of serially connected stages. The output of one stage is the input to the next stage of the decision making and can have great influence on the subsequent stages. Often the parameters of one stage act as constraints to another stage, thus restricting the design space. In the block diagram in Figure 1.2, there are I stages in the life cycle; each stage i being represented by a set of *state variables* (or input parameters) $\boldsymbol{S}_{i-1}$, a set of stage decision variables (or design variables) $\boldsymbol{X}_i$, a set of transformation functions F_i^*, an objective function U_i, and a set of output parameters $\boldsymbol{S}_i$.

The stage behavioral model for each stage is described by its transformation equations that map the stage input parameters and stage decision variables to the stage output parameters. The stage i receives input parameters $\boldsymbol{S}_{i-1}$ in conjunction with stage decision variables $\boldsymbol{X}_i$ and it produces the stage output $\boldsymbol{S}_i$. In general, this relationship is described by a set of transformation equations and can be expressed as

$$F^{*}_{i}(\boldsymbol{S}_i, \boldsymbol{S}_{i-1}, \boldsymbol{X}_i) = \mathbf{0}, \quad i=1,..,I \tag{5.1}$$

If the transformation function can be explicitly expressed, then the transformation equations for stage i can be written as

$$\boldsymbol{S}_i = \boldsymbol{F}_i(\boldsymbol{S}_{i-1}, \boldsymbol{X}_i), \tag{5.2a}$$

or the relationship for j'th output of stage i can be expressed as

$$S_{ij} = F_{ij}(\boldsymbol{S}_{i-1}, \boldsymbol{X}_i), \quad \text{for } j=1,..,J_i \tag{5.2b}$$

where J_i is the number of transformation equations (and output variables) for stage i. The objective function at each stage is the cost component associated with that stage. For example, the design stage objective function is the cost of R&D and product development, and the operation stage objective function is the cost of operation, support and maintenance for the life of the product. The stage objective function can be expressed as

$$U_i = U_i(\boldsymbol{S}_{i-1}, \boldsymbol{X}_i) \tag{5.3}$$

Optionally, a stage decision could be constrained by its given input and required output. These constraints could be expressed as

$$G_{im} = G_{im}(\boldsymbol{S}_{i-1}, \boldsymbol{X}_i, \boldsymbol{S}_i) \leq 0, \quad \text{for } m=1,...,M \tag{5.4}$$

The transformation equations, the objective functions, and the constraints could be of simple closed form, or may require numerical simulation, or may be empirical charts. Finally, the overall objective function for all the stages combined is the life-cycle cost (LCC), which can be expressed as

$$U = U(\boldsymbol{S_0}, \boldsymbol{X_1},...,\boldsymbol{X_i},..., \boldsymbol{X_I}), \quad \text{i=1,..,I} \tag{5.5}$$

In the case of life-cycle optimization, the overal objective function is additive

$$U = \sum U_i(\boldsymbol{S_{i-1}}, \boldsymbol{X_i}), \qquad \text{i=1,...,I} \tag{5.6}$$

Though the problem of life-cycle cost optimization has been described as serial, the stages are highly coupled in nature. A special method which can be used to decompose (uncouple) the serial problem is the use of dynamic programming. The basic concept of dynamic programming is known as Bellman's principle of optimality [Bell57]:

> An optimal sequence of decisions in a multistage decision process problem has the property that whatever the initial stage, state and decisions are, the remaining decisions must constitute an optimal sequence of decisions for the remaining problem, with the stage and state resulting from the first decision considered as initial conditions.

The following is the general formulation of dynamic programming problem. A system with I discrete stages can be represented by a cascade block diagram as shown in

Figures 1.2 and 1.3. The design vector (decision variables) and the input vector (state variables) for ith stage can be represented as,

$$X_i = \{X_{i,1},\ldots, X_{i,n},\ldots, X_{i,Ni}\}, \tag{5.7}$$

$$S_{i-1} = \{S_{i-1,1}, \ldots, S_{i-1,j}, \ldots, S_{i-1,Ji}\}, \tag{5.8}$$

where N_i is the total number of design variables, J_i is the total number of state variables for the i'th stage.

The dynamic programming solution procedure is started with the last stage and applied at each stage going backward until the first stage. The optimum values of the decision variables at a stage depend on the values of the state variables at that stage and the effect of the stage decision on subsequent stages. We define an *optimal value function* for stage i, $R_i(S_{i-1})$, which represents the optimized value of the combined objective function from stage i to the last stage I. Evaluation of optimal value function assumes that optimum decisions will be made in stage i and all the subsequent stages. In other words, $R_i(S_{i-1})$ is a function denoting the optimum value of the objective function of a dynamic programming problem consisting of stages i to I with the input to stage i, being given by vector S_{i-1}. Therefore optimum value function at a stage is only a function of the state variable at that stage

$$R_i(S_{i-1}) = \underset{X_i, X_{i+1}, \ldots X_I}{\text{Min}} \sum_{i}^{I} U_i(S_{i-1}, X_i) \tag{5.9}$$

The selection of values of the design variables X_i for each stage i and initial state S_{i-1} is based on the optimal objective function from the final stage upto i'th stage. Now we define a function Q_i, which is the sum of the objective function for stage i and the optimum value function for stage $i+1$, as follows

$$Q_i(S_{i-1}, X_i, S_i) = U_i(S_{i-1}, X_i) + R_{i+1}(S_i) \tag{5.10a}$$

Recognizing from equations 5.1 or 5.2 that S_i is a function of S_{i-1} and X_i, the above relationship can be expressed as

$$Q_i(S_{i-1}, X_i) = U_i(S_{i-1}, X_i) + R_{i+1}(F_i(S_{i-1}, X_i)) \tag{5.10b}$$

which is the sum of objective function at stage i and the optimum value function for stage $i+1$. Thus Q_i is a function of the input state variables and the decision variables for that stage. The optimum value function R_i for stage i, for a given (fixed) set of state variables S_i, is obtained by minimizing Q_i as a function of decision variables X_i. Hence minimum value function R_i itself is a function of state variables at stage i and can be expressed as

$$R_i(S_{i-1}) = \underset{X_i}{\text{Min}}\ Q_i(S_{i-1}, X_i) \tag{5.11}$$

Now the basic *dynamic programming recursive relationship* controlling the selection of values for design variables at i'th stage is obtained as follows

$$R_i(\boldsymbol{S}_{i-1}) = \underset{\boldsymbol{X}_i}{\text{Min}}\ [U_i(\boldsymbol{S}_{i-1}, \boldsymbol{X}_i)] + R_{i+1}(\boldsymbol{S}_i) \tag{5.12}$$

The problem in equation 5.12 consists of choosing optimum values ($\boldsymbol{X}'_i$) for the decision variables such that the combined objective function from stage i to I is optimized. Thus the recursive dynamic programming solution procedure given by equation 5.12 proceeds from the last stage backwards and ends at the first stage. At a stage i under consideration, the optimum selection of decision variables $\boldsymbol{X}_i$ depends on the value of state variables $\boldsymbol{S}_{i-1}$ (input to stage i) and the effect of optimization of the stage on the subsequent stages. Therefore, in order to make optimum decisions in equation 5.12 at stage i, we must know the value of the state variables $\boldsymbol{S}_{i-1}$ and the optimum value function for the next stage (R_{i+1}) as a function of its input ($\boldsymbol{S}_i$). Since the value of state variables $\boldsymbol{S}_{i-1}$ is not known in advance, a reasonable number of possible values within a certain range must be considered. Hence a reasonable number of dicretizations of the range of possible state variables at the beginning of each stage are considered and the corresponding objective function evaluated for each stage. Specifically, let $\boldsymbol{S}_{i,min}$ and $\boldsymbol{S}_{i,max}$ denote the range of possible values for state variables at stage i+1 as

$$\boldsymbol{S}_{i,min} = [S_{i1,min}, .. \ S_{ij,min}, .. \ S_{iJi,min}], \tag{5.13a}$$

$$\boldsymbol{S}_{i,max} = [S_{i1,max}, .. \ S_{ij,max}, .. \ S_{iJi,max}], \tag{5.13b}$$

such that the j'th variable lies within the range

$$S_{ij,min} \leq S_{ij} \leq S_{ij,max} \tag{5.14}$$

Here a reasonable Minimum set $\boldsymbol{S}_{i,min}$ and a Maximum set $\boldsymbol{S}_{i,max}$ for the state variables must be estimated in advance.

5.2 Dynamic Programming Algorithm

The dynamic programming recursive relationships described above by equation 5.12 is implemented through the tabular computations as shown in Tables 5.1 and 5.2. Table 5.1 lists important variables computed for stage i while traversing the stages backwards. After having constructed a table for each stage (total I tables), we trace back the path for overall optimum solution from stage 1 forward to stage I. Table 5.2 enumerates the important variables computed for stage i while tracing back the overall optimum solution and traversing the stages forward. Tables of type 5.1 and 5.2 will be called *recursion tables* and *trace-back tables* respectively.

5.2.1 Construction of Recursion Tables

The following is a description of tabular computations and construction of recursion table for stage i as given in Table 5.1. The description of recursion table for stage i is based on the assumption that the tabular computations for stage I through i+1 have already been performed. For stage $i+1$, each component of state variable vector $\boldsymbol{S}_i$ is linearly discretized within the range described by equations 5.13 and 5.14. If K_{ij}

Table 5.1: Recursion Table. Recursive tabular computations of different variables for stage i in dynamic programming optimization.

Different Discreti-zations at Stage i (1)	State Variable Discreti-zations (2)	Optimum Stage Decision-Variable Vector for Stage i (3)	Optimum Stage Objective-Function for Stage i (4)	Resulting State Variable Vector for Stage $i+1$ (Output of Stage i) (5)	Optimum Value Function for Stage $i+1$ (6)	Objective Function for Stage i to I Combined (7)=(4)+(6)
1	$S_{i-1,1}$	$X'_{i,1}$	$U'_{i,1}$	$S'_{i,1}$	$R_{i+1}(S'_{i,1})$	$R_{i,1}$
:	:	:	:	:	:	:
k	$S_{i-1,k}$	$X'_{i,k}$	$U'_{i,k}$	$S'_{i,k}$	$R_{i+1}(S'_{i,k})$	$R_{i,k}$
:	:	:	:	:	:	:
K_i	$S_{i-1,Ki}$	$X'_{n,Ki}$	$U'_{i,Ki}$	$S'_{i,Ki}$	$R_{i+1}(S'_{i,Ki})$	$R_{i,Ki}$

such discretizations are done for the j'th component of $\boldsymbol{S}_i$ within the range ($S_{ij,min}$, $S_{ij,max}$), then the j'th discretization is given by

$$S_{ij,k} = S_{ij,\min} + \frac{S_{ij,\max} - S_{ij,\min}}{K_{ij} - 1}\,(k-1), \quad k = 1,..,K_{ij} \tag{5.15}$$

and the total number of the combinations P_i of such discretization is given by,

$$P_i = \prod_{j=1}^{J_i} K_{ij} \tag{5.16}$$

Hence the recursion table shows P_i such discretizations of the state variable vector at stage $i+1$. For stage i, the state variable vector discretizations $\boldsymbol{S}_{i-1,p}$, given in column (2), are used as input to the stage for evaluating the partial objective function $U_i(\boldsymbol{S}_{i-1,p}, X_i)$. The function Q_i described by equation 5.10 is used as the full objective function for local stage optimization. For the p'th dicretization, it can be expressed as

$$\text{Min } Q_{i,p}(\boldsymbol{S}_{i-1,p}, X_i) = \text{Min } [\ U_i(\boldsymbol{S}_{i-1,p}, X_i) + R_{i+1,p}(\boldsymbol{F}_i(\boldsymbol{S}_{i-1,p}, X_i))] \tag{5.17}$$

The crucial step now is to evaluate the value of optimum value function for stage i+1, which is the second term on the right hand side in equation 5.17, and the column (6) of the recursion table. After computing $\boldsymbol{S}_i$ in column (5) (or $\boldsymbol{F}_i(\boldsymbol{S}_{i-1,p}, X_i)$), the corresponding optimum value function $R_{i+1,p}$ is computed using already computed recursion table for stage i+1. To do this, state variable vectors $\boldsymbol{S}_i$ given in column (2)

and the corresponding optimum value function R_{i+1} given in column (7) of the recursion table for stage i+1 are used. The recursion tables for stage i+1 would only contain discrete values of $\boldsymbol{S}_i$ vectors in column (2), which, in general, will not match the desired vector $\boldsymbol{S}_{i+1}$ in column (5) of the recursion table for stage i. Thus, in order to compute the optimum value function for stage i+1, a general multi-dimensional interpolation scheme is used and is stated in the following paragraphs.

5.2.2 Multi-Dimensional Interpolation

The multi-dimensional interpolation procedure, which was implemented for a very general case, uses quadratic and cubic interpolations. The multi-dimensional interpolation problem can be stated as follows. Let v be a function of a J dimensional vector $\boldsymbol{u}$ defined by some unknown function ϕ,

$$v = \phi(\boldsymbol{u}) \tag{5.18}$$

where

$$\boldsymbol{u} = \{u_1, u_2, \ldots u_J\}. \tag{5.19}$$

Let all discretizations of j'th component, u_j, be stored in vector $\overline{\boldsymbol{u}}_j$ as follows

$$\overline{\boldsymbol{u}}_j = \{\overline{u}_{j,1}, \ldots \overline{u}_{j,k}, \ldots \overline{u}_{j,Kj}\}, \quad \text{for } j=1,\ldots J \tag{5.20}$$

which are stored in a $P \times J$ matrix $\overline{\boldsymbol{U}}$ as row vectors. The corresponding function values (at nodal point p) v_p are stored in a vector $\boldsymbol{V}$, such that

$$v_p = \phi(\boldsymbol{u}_p), \quad \text{for } p=1,..P. \tag{5.21}$$

In the interpolation problem encountered in the general dynamic programming problem, matrices $\overline{\boldsymbol{U}}$ and $\boldsymbol{U}$, and vector $\boldsymbol{V}$ are known. The objective is to evaluate the unknown function $v^*=\phi(\boldsymbol{u}^*)$ for a given vector $\boldsymbol{u}^*$ using interpolation of surrounding nodal points in J dimensional space. In this thesis, both quadratic and cubic polynomial multi-dimensional interpolations were implemented and are described in Appendix A.

When the recursion table are constructed while traversing backwards through the stages, the interpolations basically map the columns (2) and (7) of stage i+1 onto the columns (5) and (6) of stage i. According to equation 5.12, the optimum value function for stage i, in column (7) of the recursion table, is obtained by simply adding column (4) and (6). R_i in column (7) also serves as the objective function at stage i. Since the input state variables $\boldsymbol{S}_o$ for stage 1 are prescribed for the problem, no discretizations for state variable vector are needed. Hence the recursion table for stage 1 will always have only one row in it (i.e. K_l=J_l=1).

5.2.3 Stage Optimization

At each stage of the dynamic programming solution approach, we encounter the optimization problem stated by equations 5.10 to 5.12. The optimization is carried out subject to the M_i inequality constraints given by equations 5.4. The optimization problem for stage i, with decision variables being $\boldsymbol{X}_i$ and the parameters being $\boldsymbol{S}_{i-1,p}$, is restated as follows

minimize $$Q_{i,p}(S_{i-1,p}, X_i) = U_i(S_{i-1,p}, X_i) + R_{i+1,p}(F_i(S_{i-1,p}, X_i)) \tag{5.23}$$

subject to $$G_{i,m} = G_{i,m}(S_{i-1,p}, X_i, S_i) \leq 0, \text{ for } m=1,..,M_i \tag{5.24}$$

where, from equations 5.7 and 5.8, decision variable vector and state variable vector respectively are

$$X_i = \{X_{i,1}, .. X_{i,n}, .. X_{i,Ni}\}, \tag{5.25}$$

$$S_{i-1,p} = \{S_{i-1,p,1}, .. S_{i-1,p,j}, .. S_{i-1,p,Ji}\}, \tag{5.26}$$

where N_i is the total number of design variables, J_i is the total number of state variables, and M_i is the number of constraints for the i'th stage. The stage optimization problem in equations 5.23 - 5.26 is stated for the p'th combination of the state variable vector (nodal point p), with p=1,..P_i. This is a general constrained non-linear optimization problem and since it is also a part of multi-level optimization approach, it will be discussed in detail in the following chapter.

5.2.4 Construction of Trace-Back Tables

After the backward recursive traversal procedure has been completed, and as a result, the recursion tables have been constructed, the overall optimum solution is obtained by tracing back the existing recursion tables in the forward direction from the first stage through the last stage. At stage 1, the state variables (input vector S_o) is known in advance, hence the recursion table contains only one row (since there are no state variable discretizations needed). At stage 1, the optimum decision variables X'_1

Table 5.2: Trace-Back Table. Tabular computations of different variables at optimum points at stage i while tracing back the overall optimum solution in dynamic programming approach.

Discretization at Stage i (1)	Optimum State Variable (2)	Overall Optimum Stage Decision-Variable Vector (3)	Optimum Stage Objective-Function (4)	Resulting State Variable Vector for Stage $i+1$ (Output of Stage i) (5)	Optimum Value Function for Stage $i+1$ (6)	Optimum Objective Function for Stage i to I Combined (7)=(4)+(6)
1	S^*_{i-1}	X^*_i	U^*_i	S^*_i	$R_{i+1}(S^*_i)$	R^*_i

in column (3) of recursion table (Table 5.1) are the overall optimum decisions, $\boldsymbol{X}^*_1$, since they lead to overall optimum. The selection of $\boldsymbol{X}^*_1$ in stage 1 leads to optimum state variables $\boldsymbol{S}^*_1$ at stage 2. Subsequently the stage optimization of equation 5.10 for stage 2, with $\boldsymbol{S}^*_1$ as input, will lead to overall optimum decisions, $\boldsymbol{X}^*_2$, and optimum state variables $\boldsymbol{S}^*_2$ at stage 3. In general, for a stage i we would know the optimum state variables $\boldsymbol{S}^*_{i-1}$, and the stage optimization will lead to overall optimum decisions, $\boldsymbol{X}^*_i$, and optimum state variables $\boldsymbol{S}^*_i$. As discussed earlier, the stage optimization problem in equations 5.23 - 5.24 would require the optimum value function for stage i+1 ($\boldsymbol{R}_{i+1}$). In general, the state variables $\boldsymbol{S}^*_i$ evaluated (needed to evaluate $\boldsymbol{R}_{i+1}$) will not be found in recursion table for stage i. To obtain the optimum value function $\boldsymbol{R}_{i+1}$, multi-dimensional interpolation is again invoked. Note that while tracing back the optimum solution, optimization is done only for one set of state variables at each stage, as against P_i sets of optimization needed while constructing the recursion tables.

This process of optimization for stage i is explained by constructing the *trace-back table* for each stage as described by Table 5.2. Column (2) of the trace-back table contains the output state variable $\boldsymbol{S}_{i-1}$ from the previous stage (i-1). After stage optimization, the optimum decision variables $\boldsymbol{X}^*_i$ are entered in column (3); the optimum objective function $\boldsymbol{U}^*_i$ is entered in column (4); the resulting output state variables $\boldsymbol{S}^*_i$ are entered in column (5); the corresponding optimum value function for the next stage R_{i+1}, obtained by interpolation, is entered in column (6). Column (7) shows the sum of column (4) and (6). By tracing back in this manner, the over all optimum solution to the dynamic programming problem is obtained. The optimum

value function for stage i+1 (R_{i+1} in column (6) of Table 5.2) is evaluated by multi-dimensional interpolation as described in detail in Appendix A.

```
Procedure 6.1: Discretize
{
    input J_o
    input vector S_o[1..J_o];
    for stage i=1 to I {
        input number of state variables J_i for stage i;
        input vectors S_i,min[1..J_i], S_i,max[1..J_i] for stage i;
        input number of discretizations L_i at stage i;
        for j=1 to J_i
            create discretization points S_i,j;
        evaluate K_i;
        /* Create discretized nodal points as follows. */
        k=1;
        k_index[1..J_i] = ConvBase(k, L_i, J_i);
        for j=1 to J_i
            S'_i,k,j = S̄_i,j,[k_index[j]];
        for nodal-point k=2 to K_i
        {
            k_index[1..J_i] = Add1(L_i, k_index);
            for j=1 to J_i
                /* define the nodal points matrix S' as follows. */
                S'_i,k,j = S̄_i,j,[k_index[j]];
        };
    };
};

Procedure 6.2: DynamicProg
{
    call Discretize;
    for stage i=1 to I
    {
        for nodal-point k=1 to K_i
        {
            Optimize_ND_Cnstr (Function_Q_i, i, S̄_i,k, M_i, x^o, Q_i, Q_e, X'_i,k, R'_i,k
                AL, Q_ik, Si-1_i,k, Fi_1_i,k, CV);
            /* Where the input and output to the procedure are as follows:
                Input:
                    function_Q_i    Procedure to evaluate Q_i.
```

i	Stage.
$\overline{S}_{i,k}[1..J_i]$	Nodal point k for state variable vector.
M_i	Number of decision variables at stage i.
$X^o[1..M_i]$	Initial value of decision variable vector at stage i.
$Q_{i,i}$	Number of inequality constraints at stage i.
$Q_{i,e}$	Number of equality constraints at stage i.

Ouput:

$X'_{i,k}[1..J_i]$	Resulting optimum decision variable vector at stage i for k'th nodal point.
$R'_{i,k}$	Resulting optimum-value-function for stage i for k'th nodal point.
AL	Augmented Lagrangian function.
Q_{ik}	
$Si\text{-}1_{i,k}$	
$Fi\_1_{i,k}$	
CV	Constraints violations array. The array contains the list of constraints (constraint numbers) violated at the optimum point.

*/

};

Procedure 6.3: *Optimize_ND_Cnstr (function_F, i,* $\overline{S}$, M_i, X^o, Q_i, Q_e, $X'_{i,k}$, $R'_{i,k}$, *AL,* Q_{ik}, $Si\text{-}1_{i,k}$, $Fi\_1_{i,k}$, *CV)*

{

Procedure Input:

function_Q_i	Procedure to evaluate Q_i.
i	Dynamic programming stage number.
J	Number of parameters (dimensionality of S).
$\overline{S}[1..J]$	Parameter variable vector.
N	Number of decision variables (dimensionality of X).
$X^o[1..N]$	Initial value of decision variable vector.
Q_I	Number of inequality constraints.
Q_e	Number of equality constraints.

Procedure Ouput:

$X^*[1..N]$	Resulting optimum decision variable vector at stage i for k'th nodal point.
R^*	Resulting value of optimum function (F).
AL	Augmented Lagrangian function = R^* + P^*; Where P^* is the penalty function at the optimum.

Q_i	Value of optimum-value-function for stage i (combined optimum objective function from stage i to I).
$nextS_i$	Output state variables vector (stage i+1 input).
$nextQ_i$	Value of optimum-value-function for stage i+1 (combined optimum objective function from stage i+1 to I).
CV	Constraints violations array. The array contains the list of constraints (constraint numbers) violated at the optimum point.

Procedure Approach:

The procedure uses a constrained non-linear optimization approach to optimize a given function of decision variable vector $\boldsymbol{X}[1..N]$ and parameters vectors $\boldsymbol{S}[1..J]$.

};

Procedure 6.4: *ConvBase*(k_{10}, *b, m*)

{

Given a number k_{10} in decimal base, convert it into base b. It returns an array k_b[1..m], of length m, containing the converted number.

};

Procedure 6.5: *Add1*(*b, m,* k_b)

{

Given a number k_b in base b (an array of length m), increment the number by 1 (add 1) and return it (an array k_b[1..m], of length m.

};

CHAPTER 6
THE INTEGRATED DESIGN DECISION SYSTEM

For the life-cycle optimization, three different solution approaches are investigated and implemented. The first two approaches tried are algorithmic and the third approach combines an algorithmic approach with a heuristic approach. The algorithmic optimization approaches implemented are very general in nature.

The first optimization approach, as presented in the previous chapter, is to treat the life-cycle optimization as a dynamic programming problem. A very general nonlinear constrained optimization module was developed which can handle a design vector of arbitrary size and any number of constraints. The constrained optimization problem is first converted into an unconstrained optimization problem by adding a combination of interior and exterior penalty functions to the objective function, and then using augmented Lagrange multiplier method. The unconstrained problem is optimized using conjugate directions or variable metric methods.

The second optimization strategy is a multi-level optimization approach, where the design optimization is decomposed into many levels. Here the design vector is decomposed into a global design vector and level-specific design vectors. At each level of optimization, calls to nonlinear constrained optimization modules are made.

The third optimization approach is a combination of heuristic and algorithmic procedures. Generally, the combination of the two procedures in a decision system can provide the best design tools. The heuristic system generates an initial design based on the application domain of the bearing specifications supplied by the designer. The heuristic module can also process the feedback about constraint violations and modify the appropriate parameters so that the design remains within the feasible domain.

6.1 Multilevel Optimization Approach

A general discussion of multilevel optimization now follows. While the well known nonlinear optimization techniques have proved effective where limited number of variables and constraints are involved, large number of variables may require excessive computer time. One remedy for this problem lies in the resolution of the large problem into smaller units. One approach, presented in the last chapter, is the dynamic programming method, where the problem is treated in stages with optimal decisions being made at each stage. Yet another approach to this problem is based on decomposition of the system into a number of smaller subsystems, each with its own objective function and constraints, solved independently at the 1st level and coordinated at the 2nd level. General formulation of such decomposition, based on Kirsch [Kirs75], and Kirsch and Moses [Kirs79] is presented in this sections.

Consider the following general optimization problem with decision variables being the vector $\boldsymbol{W}$

Minimize $$Z = f(W), \tag{6.1}$$

Subject to constraints

$$\mathbf{h}(W) = \mathbf{0} \tag{6.2}$$

$$\mathbf{g}(W) \leq \mathbf{0} \tag{6.3}$$

$$W^L \leq W \leq W^U \tag{6.4}$$

where W^L are the lower bounds and W^U are the upper bound for W. The multilevel optimization approach consists of decomposition of this general problem into subproblems, in which so-called coordination variables are chosen by second level controller to ensure solutions for the independent first-level system that actually correspond to an overall system optimum. As would be expected, success is conditional on suitable conversion of the integrated problem to the two level form, in which the first-level result must have noninteracting parts capable of separate treatment. In the model coordination approach, the vector W is partitioned into subvectors, $\overline{Y}$ and $\overline{X}$

$$W = \begin{bmatrix} \overline{Y} \\ \overline{X} \end{bmatrix} \tag{6.5}$$

$$\overline{X} = \begin{bmatrix} \overline{X}_1 \\ \vdots \\ \overline{X}_n \\ \vdots \\ \overline{X}_N \end{bmatrix} \tag{6.6}$$

in which $\overline{Y}$ is the subvector of interactions or coordination variables between the subsystems, and $\overline{X}$ is the subvector of variables for subsystem, with vector $\overline{X}_n$ being the vector for n'th subsystem. The partitioning of the decision variable vector W is done in such a way so that the equality and inequality constraints and the objective function can be rewritten in the form

$$\mathrm{h}(W) = \begin{bmatrix} \mathrm{h}_1(\overline{Y},\overline{X}_1) \\ \vdots \\ \mathrm{h}_n(\overline{Y},\overline{X}_n) \\ \vdots \\ \mathrm{h}_N(\overline{Y},\overline{X}_N) \end{bmatrix} \tag{6.7}$$

$$\mathrm{g}(W) = \begin{bmatrix} \mathrm{g}_1(\overline{Y},\overline{X}_1) \\ \vdots \\ \mathrm{g}_n(\overline{Y},\overline{X}_n) \\ \vdots \\ \mathrm{g}_N(\overline{Y},\overline{X}_N) \end{bmatrix} \tag{6.8}$$

and the objective function can be written in the form

$$Z = \mathrm{f}(W) = \sum_{n=1}^{N} \mathrm{f}_n(\overline{Y},\overline{X}_n) \tag{6.9}$$

Based on these assumptions, the two-level optimization problem can now be stated as follows.

First level optimization — Determine a fixed value for $\overline{Y}$ through the constraints

$$\overline{Y} = \overline{Y}^o \tag{6.10}$$

Then the optimization problem in equation 6.9 can be decomposed into N independent first-level problems, each of which is stated as follows. Find $\overline{X}_n$ such that:

$$\text{Minimize} \qquad Z_n = f_n(\overline{Y}^o, \overline{X}_n) \tag{6.11}$$

Subject to constraints

$$\mathbf{h}_n(\overline{Y}^o, \overline{X}_n) = \mathbf{0}, \quad n=1,\ldots,N \tag{6.12}$$

$$\mathbf{g}_n(\overline{Y}^o, \overline{X}_n) \leq \mathbf{0}, \quad n=1,\ldots,N \tag{6.13}$$

$$X^L \leq X \leq X^U \tag{6.14}$$

If we introduce the function

$$H_n(\overline{Y}^o) = \min Z_n \tag{6.15}$$

then the first-level problem consists of finding the function

$$H(\overline{Y}^o) = \sum_{n=1}^{N} H_n(\overline{Y}^o) = \sum_{n=1}^{N} \min Z_n \tag{6.16}$$

for all $\overline{X}_n$ (n=1,...,N) satisfying the constraints given in equations 6.12 to 6.14.

Second Level Optimization — The task in the second level problem is to find $\overline{Y}^o$, which minimizes

$$H(\overline{Y}^o) = \sum_{n=1}^{N} H_n(\overline{Y}^o) \tag{6.17}$$

Now the two level optimization problem can be solved iteratively by the following procedure (see Figure 6.1):

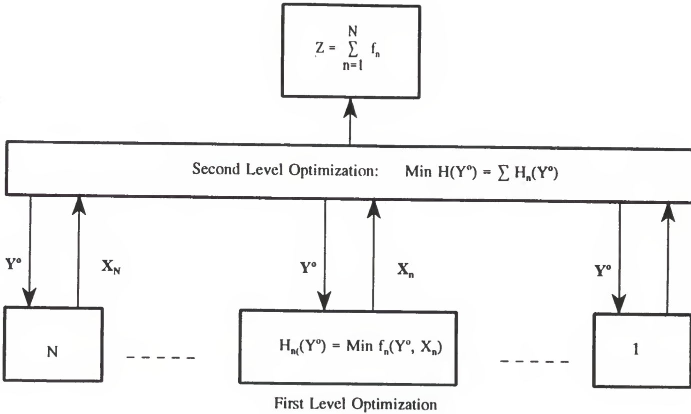


Figure 6.1: A schematic diagram for multilevel optimization.

1. Choose an initial value for the coordination variable, $\overline{Y}^o$.

2. For a given $\overline{Y}^o$, solve the N independent first-level problems.

3. Modify the value of $\overline{Y}^o$ so that the value of $H(\overline{Y}^o)$ is reduced.

4. Repeat steps 2 and 3 until minimum $H(\overline{Y}^o)$ is achieved.

The multilevel optimization approach formulated in this section is applied for the optimum design of journal bearings in the next section.

6.2 Parameter and Tolerance Design By Multilevel Optimization

Minimizing the life cycle cost is a coupled, non-linear constrained optimization problem. The life cycle cost consists of manufacturing cost, operation and maintenance cost, internal and external quality cost. The objective is to minimize the total cost rate (life-cycle cost per hour of bearing operation), which can be defined as follows

$$C_r = C_{lc}/t_l = (C_M + C_O + C_{IQ} + C_{EQ})/t_l \tag{6.18}$$

where

$$\text{manufacturing cost} = C_M = C_M(L, D, \Delta_D), \tag{6.19}$$

$$\text{operation cost} = C_O = C_O(L, D, C, \mu), \tag{6.20}$$

$$\text{internal quality cost} = C_{IQ} = C_{IQ}(\Delta_D), \tag{6.21}$$

$$\text{external quality cost} = C_{EQ} = C_{EQ}\left[C_i, \Delta_D, \frac{\partial^2 C_o}{\partial C_i^2}\right] = C_{EQ}(C_i, \Delta_D, C_o'') \tag{6.22}$$

$$\text{bearing life} = t_l.$$

The optimum design is chosen subject to constraints on bearing design given by equations 2.1 to 2.8. The decision variables being bearing length L, initial clearance C_i, oil viscosity μ, and design tolerance on bearing and journal diameter Δ_D. It is a coupled problem since the external quality cost C_{EQ} is a function of double derivative of operation cost function $C_o(C_i)$.

The optimum bearing design problem is decoupled by dividing the optimization problem into two levels. The schematic diagram in Figure 6.2 presents the algorithm for the two level optimum parameter and tolerance design for the journal bearing. In the first level we make appropriate assumptions for the values of Δ_D and $C_o''(C_i)$ and optimize for L, C_i, and μ. After this optimization the operation cost curve as a function C_i is available around the optimum initial clearance C_i^*. Thus C_{eq} can be found out more accurately. Now we optimize the total life cycle cost C_{lc} as a function of tolerance Δ_D only. With this optimum value of Δ_D^*, we can again go back to first level optimization. Comparing with the notation introduced in the general formulation of multilevel optimization problem (Figure 6.1).

$$\overline{Y} = [C_o''] \tag{6.23}$$

$$\overline{X}_I = [L, D, C_i, \mu, \Delta_D] \tag{6.24}$$

$$\overline{W} = [Y,\ X_I] = [C_o'', L, D, C_i, \mu, \Delta_D] \tag{6.25}$$

The objective function is evaluated by invoking the bearing simulation, which not only evaluates the operation cost but also determines the life of the bearing.

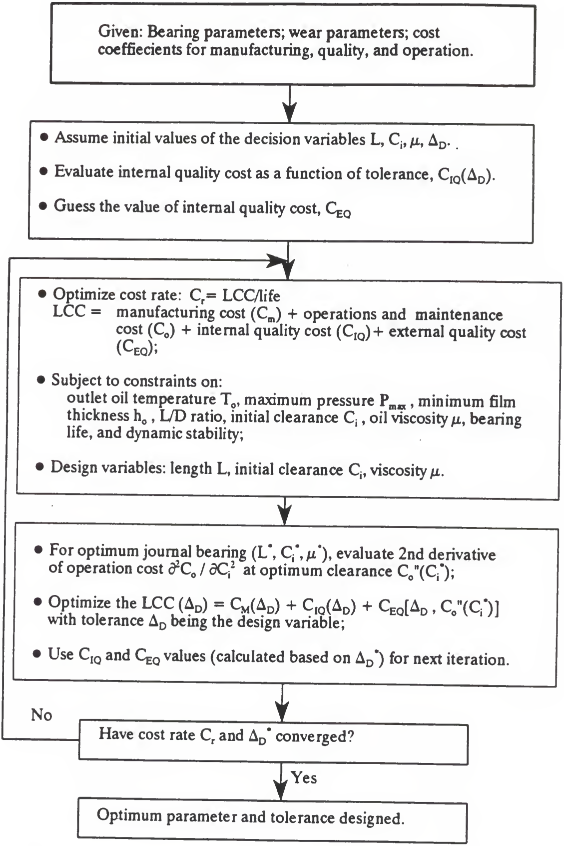


Figure 6.2: Optimum parameter and tolerance design for journal bearing.

6.3 Nonlinear Constrained Optimization

This section presents the approach used to solve the general multivariable nonlinear constrained optimization problem. This general optimization problem is encountered in selecting the optimum decision variables while using two different approaches in this thesis. First is the problem of stage optimization in the dynamic programming based approach presented in the last chapter. Second is the multilevel optimization approach discussed in the previous section. This general optimization problem is solved by converting the constrained problem to an unconstrained one, using the augmented Lagrange multiplier method. The unconstrained problem is then solved employing a conjugate directions algorithm or variable metric algorithm, with one dimensional search performed using method of golden sections. A schematic diagram illustrating the relationship between various modules of the integrated system is shown in Figure 6.3.

A brief statement of the general multivariable nonlinear constrained optimization problem now follow. Consider the general optimization problem of equations 6.1 to 6.4, restated as follows, with decision variables being the vector X.

Minimize $$Z = f(X), \tag{6.26}$$

Subject to constraints

$$h_k(X) = \mathbf{0}, \quad k=1,..,K \tag{6.27}$$

$$g_j(X) \leq \mathbf{0}, \quad j=1,..,J \tag{6.28}$$

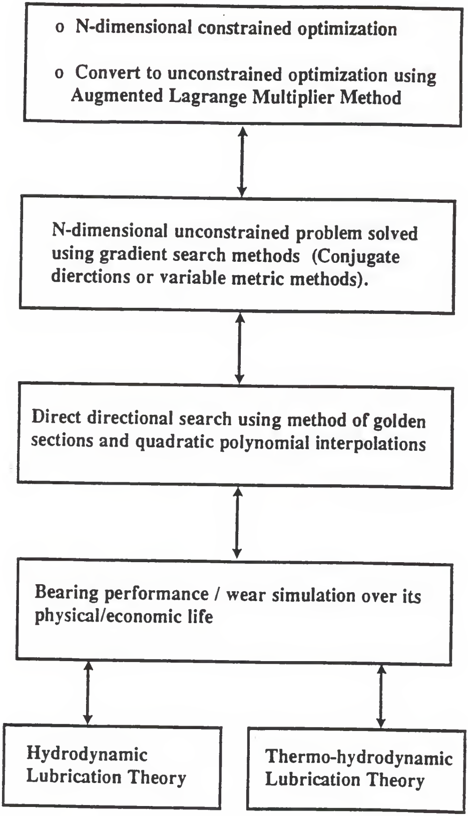


Figure 6.3: Relationship among various modules in integrated optimization system.

where the inequality constraints include the set of side constraints of type $X^L \leq X \leq X^U$. If we had only the equality constraints, the Kuhn-Tucker conditions require that if we create the Lagrangian

$$L(X,\lambda) = f(X) + \sum_{k=1}^{K} \lambda_k h_k(X) \tag{6.29}$$

then the minimization of the Lagrangian subject to equality constraints of equation 6.27 also provides the solution to the optimization problem given by equations 7.26 and 7.27. Here λ_k, for k=1,..,K are the lagrangian multipliers. Thus we can use the exterior penalty function approach to create a pseudo objective function, called augmented Lagrangian function (ALF) to be minimized as

$$A(X,\lambda,r_p) = f(X) + \sum_{k=1}^{K} \left[\lambda_k h_k(X) + r_p [h_k(X)]^2 \right] \tag{6.30}$$

An efficient method for optimization is to repeatedly optimize ALF with the λ and r_p held constant. The new set of augmented Lagrange multipliers is calculated using the following relation for updating the lagrange multiplier [Vand85]

$$\lambda_k^{p+1} = \lambda_k^p + 2r_p h_k(X^p), \quad \text{for } k=1,\dots,K \tag{6.31}$$

where p denotes the iteration number.

The optimization problem with both equality and inequality constraints can also be tackled by using the augmented Lagrangian multiplier (ALM) method. Vanderplaats [Vand85] presents the following algorithmic approach to solve the problem. The general augmented Lagrangian for an optimization problem with equality and inequality constraints is given by

$$A(X,\lambda,r_p) = f(X) + \sum_{j=1}^{J} \left[\lambda_j \psi_j + r_p \psi_j^2\right] + \sum_{k=1}^{K} \left[\lambda_k h_k(X) + r_p [h_k(X)]^2\right] \quad (6.32)$$

The update formulas for the lagrange multipliers are now given by

$$\lambda_j^{p+1} = \lambda_j^p + 2r_p \left[\max \left[g_j(X), \frac{-\lambda_j^p}{2r_p} \right] \right], \quad \text{for } j=1,\dots,J \quad (6.34)$$

$$\lambda_{k+J}^{p+1} = \lambda_{k+J}^p + 2r_p h_k(X), \quad \text{for } j=1,\dots,J \quad (6.35)$$

The general algorithm for optimization by augmented Lagrange multipliers is presented in Procedure *Optimize_ND_Constrained.* The method has been adopted to solve general nonlinear constrained optimization problems because of the following features.

1. The method is relatively insensitive to the value of the penalty multiplier r_p and the way by which they are updated. The r_p does not have to increase to ∞ for obtaining final optimum.
2. Precise satisfaction of constraints $g_j(X)=0$ and $h_k(X)=0$ is possible.
3. Acceleration towards optimum solution is obtained by updating the Lagrange multipliers.
4. The starting point may be either feasible or infeasible.
5. At the optimum, the value of the Lagrange multipliers $\lambda^*_j \neq 0$ will automatically identify the active constraint set.
6. The availability of λ^*_j at the optimum point and nearby, can facilitate the evaluation of sensitivity of optimum solution to problem parameters.

The ALF (overall life-cycle cost including the penalty) is evaluated by the Procedure *Augmented_LC_Cost*. This procedure makes calls to Procedure *MIEQ_Cost* in order to evaluate the manufacturing cost, internal quality cost and external quality cost. The Procedure *Augmented_LC_Cost* also calls the procedure *Augmented_Op_Cost* to evaluate the operation cost of the bearing.

Procedure 6.1: *Augmented_LC_Cost* (*from*, N, J, K, $\boldsymbol{X}$, C_r, V_i, V, A, λ)
{
Purpose:

Evaluate augmented Lagrange function (A) given by equation 7.32 and the cost rate (C_r = LCC/life).

Input:

The procedure which called this procedure, *from*
Number of decision variables, N;
Number of equality constraints, J;
Number of inequality constraints, K;
Decision variable vector $\boldsymbol{X}$, defined as follows:

$X_1=\Delta_D/\Delta_{D,min}$, $X_2=C_o''(C_i)$, X_3=bearing length L, X_4=bearing diameter D, $X_5=C_M$, $X_6=C_{IQ}$, $X_7=C_{EQ}$.

Output:

Life-cycle cost, C_{lc};
Life of the bearing, t_l
List of constraint violations at the beginning of life at $\boldsymbol{X}_o$, V_i;
List of constraint violations at the end of life at $\boldsymbol{X}^*$, V;
Augmented Lagrangian function, A;
Lagrange multipliers, λ.

Steps:

1. Initialize:

Initial value of Lagrange multipliers $\lambda^o=\mathbf{0}$;
Initial value of penalty multiplier r_p=1;
Coefficient by which r_p is incremented at each iteration, γ=2 to 10;
Maximum value of r_p, r_p^{max}=100 to 10000;
$\boldsymbol{X} = \boldsymbol{X}_o$;
$f = f_o = f_{LCC}(\boldsymbol{X}_o)$;

2. If LC_count = 0, then calculate the cost of manufacturing, internal and external quality by using the procedure *MIEQ_Cost*

$$MIEQ\_Cost\ (1, X, C_{MIEQ}, V_i, V)$$

3. If the current value of tolerance on clearance (Δ_D) is out of its range ($\Delta_{D,min}$, $\Delta_{D,max}$) or if it makes the clearance out of its range (C_{min}, C_{max}), then adjust the value of the tolerance as follows

$$\begin{aligned}
&\text{If } C_o\text{-}C_{min} < \Delta_D, \text{ then } \Delta_D = C_o\text{-}C_{min}\\
&\text{If } C_{max}\text{-}C_o > \Delta_D, \text{ then } \Delta_D = C_{max}\text{-}C_o\\
&\text{If } \Delta_D < \Delta_{D,min}, \text{ then } \Delta_D = \Delta_{D,min}\\
&\text{If } \Delta_D > \Delta_{D,max}, \text{ then } \Delta_D = \Delta_{D,max}
\end{aligned}$$

4. Evaluate three different values of initial clearance, $C_{i,a}=C_o\text{-}\Delta_D$, $C_{i,b}=C_o$, $C_{i,c}=C_o+\Delta_D$. Initialize 3 decision variable vectors X_a, X_b, and X_c, such that $X_{a,2}=C_{i,a}/C_{i,min}$, $X_{b,2}=C_{i,b}/C_{i,min}$, $X_{c,2}=C_{i,c}/C_{i,min}$. Now evaluate the bearing operation cost-rate for them as follows

$$\begin{aligned}
&Operation_Cost\ (N, X_a, C_{r,a}, C_{O,a}, C_{oil,a}, C_{energy,a}, t_{l,a}, V_{i,a}, V_a),\\
&Operation_Cost\ (N, X_b, C_{r,b}, C_{O,b}, C_{oil,a}, C_{energy,a}, t_{l,b}, V_{i,b}, V_b),\\
&Operation_Cost\ (N, X_c, C_{r,c}, C_{O,c}, C_{oil,a}, C_{energy,a}, t_{l,c}, V_{i,c}, V_c).
\end{aligned}$$

Evaluate the operation cost rate at these three points

$$\begin{aligned}
C_{Or,a} &= (C_{oil,a} + C_{energy,a})/t_{la}\\
C_{Or,b} &= (C_{oil,b} + C_{energy,b})/t_{lb}\\
C_{Or,c} &= (C_{oil,c} + C_{energy,c})/t_{lc}
\end{aligned}$$

Given 3 points for initial-clearance versus operation-cost-rate, $(C_{i,a}, C_{Or,a})$, $(C_{i,b}, C_{Or,b})$ and $(C_{i,c}, C_{Or,c})$, find the second derivative of cost-rate C_{Or} with respect to clearance C_i (C_{Or}'') assuming a quadratic polynomial.

5. Evaluate the gradient of C_{MIEQ} with respect to tolerance Δ_D and then the search direction

$$s = -\ \text{gradient}/\ |\ \text{gradient}\ |\ .$$

Do a 1 dimensional optimization with respect to tolerance Δ_D in the direction s using the procedure,

$$Optimize\_1D\ (MIEQ\_Cost(),\ Augmented\_LCCost,\ 7, X_b, f, s, \alpha_{max}, \alpha^*, X^*, f^*, R, V_i, V)$$

6. If the optimum tolerance Δ_D^* makes $(C_i \pm \Delta_D^*)$ go outside the specified range of clearance ($C_i min$, $C_{i,max}$), then bring it back within this range by adjusting Δ_D^* so that

$$C_{i,min} \leq (C_i - \Delta_D^*) \text{ and } C_{i,max} \geq (C_i + \Delta_D^*)$$

7. Evaluate the Augmented Lagrange function using the procedure

called_from = Augmented_LC_Cost
ALF = Augmented_Op_Cost (called_from, N, J, K, x, C_r, V_i, V)

8. Exit.

}

Procedure 6.2: *MIEQ_Cost (called_from, N, X, C_{MIEQ}, t_l, V_i, V)*
{
Purpose:
Evaluate manufacturing cost (C_m), internal quality cost (C_{iq}), and external quality cost (C_{EQ}).
Input:
The procedure which called this procedure, *called_from*
Number of decision variables, N=1 (or 7, if called_from = *Optimize_1D*);
Decision variables vector, X:
$X_1=\Delta_D/\Delta_{D,min}$, $X_2=C_o''(C_i)$, X_3=bearing length L, X_4=bearing diameter D

Output:
Decision variables vector, X:
$X_1=\Delta_D/\Delta_{D,min}$, $X_2=C_o''(C_i)$, X_3=bearing length L, X_4=bearing diameter D, $X_5=C_M$, $X_6=C_{IQ}$, $X_7=C_{EQ}$
Manufacturing + internal quality + external quality costs, C_{MIEQ};
Bearing life, t_l
List of constraint violations at the beginning of life at X_o, V_i;
List of constraint violations at the end of life at X^*, V;
}

Procedure 6.3: *Augmented_Op_Cost (called_from, N, X, C_O, t_l, V_i, V)*
{
Purpose:

Evaluate manufacturing cost (C_m), internal quality cost (C_{iq}), and external quality cost (C_{EQ}).

Input:

The procedure which called this procedure, *called_from*
Number of decision variables, N=1 (or 7, if called_from = *Optimize_1D*);
Decision variables vector, X:

$X_1=\Delta_D/\Delta_{D,min}$, $X_2=C_o''(C_i)$, X_3=bearing length L, X_4=bearing diameter D

Output:

Decision variables vector, X:

$X_1=\Delta_D/\Delta_{D,mm}$, $X_2=C_o''(C_i)$, X_3=bearing length L, X_4=bearing diameter D,
$X_5=C_M$, $X_6=C_{IQ}$, $X_7=C_{EQ}$

Manufacturing + internal quality + external quality costs, C_{MIEQ};
Bearing life, t_l
List of constraint violations at the beginning of life at X_o, V_i;
List of constraint violations at the end of life at X^*, V;
}

Procedure 6.4: *Optimize_ND_Constrained (Augmented_LCC(), N, J, K, X^o, f^o, X^*, f, V_i, V, A, λ)*
{
Purpose:

Perform constrained optimization of a function of X (a N dimensional vector).

Input:

Augmented Life-cycle cost function to be optimized, A_{LCC};
Number of decision variables, N;
Number of equality constraints, J;
Number of inequality constraints, K;
Initial values for decision variable vector X^o.

Output:

Values of the function at X^o, f^o;
Optimum values for decision variable vector X^*;
Optimum cost function, f^*;
List of constraint violations at the beginning of life at X_o, V_i;

List of constraint violations at the end of life at X^*, V;
Augmented Lagrangian function, A;
Lagrange multipliers, λ.

Steps:

1. Initialize:
   Initial value of Lagrange multipliers λ^o=$\mathbf{0}$;
   Initial value of penalty multiplier r_p=1;
   Coefficient by which r_p is incremented at each iteration, γ=2 to 10;
   Maximum value of r_p, r_p^{max}=100 to 10000;
   $X = X_o$;
   $f = f_o = f_{LCC}(X_o)$;

2. Minimize augmented Lagrange function (ALF) $A(X,\lambda,r_p)$ as an unconstrained function using the conjugate directions method (Procedure *Optimize_ND_CD*) or using the variable metric method (Procedure *Optimize_ND_VM*) as follows:
   Optimize_ND_CD $(A_{LCC}, N, J, K, X, f, X^*, f^*, V_i, V, A)$,
   or
   Optimize_ND_VM $(A_{LCC}, N, J, K, X, f, X^*, f^*, V_i, V, A)$

3. Check if the optimum solution has converged, using

   $$|(f^*-f)|/f < \varepsilon; \quad g_j(X^*) < 0; \quad |h_k(X^*)| < \varepsilon_1$$

   where, ε and ε_1 are small numbers.

4. Exit, if solution has converged.

5. Update the Lagrange multipliers using

   $$\lambda_j = \lambda_j + 2r_p \max[g_j(X^*), -\lambda_j/2r_p], \quad j=1,..,J$$

   $$\lambda_{k+J} = \lambda_{k+J} + 2r_p\, h_k(X^*), \quad k=1,..,K$$

6. Update the penalty multiplier using, $r_p = \gamma r_p$.
   If $r_p > r_p^{max}$, make $r_p = r_p^{max}$.

7. $f = f^*$

8. Goto step 2.

}

6.4 Unconstrained Optimization

Once a constrained optimization problem has been converted into an unconstrained problem by introducing augmented Lagrange function, it is optimized by using

- Fletcher and Reeves conjugate directions method [Flet64], or
- variable metric methods — specifically, Broydon-Fletcher-Goldfarb-Shanno (BFGS) method [Huan70].

These two unconstrained optimization methods were implemented and are briefly described in the following subsections.

6.4.1 Conjugate Directions Method

In this method, in the initial search direction in the first iteration is taken to be steepest descent (s=-∇f). On the subsequent iterations (say iteration number q), the search direction is modified to be a conjugate direction as follows

$$\boldsymbol{s}_q = -\nabla f(\boldsymbol{X}_q) + \beta_q \boldsymbol{s}_{q-1} \tag{6.36}$$

where the scalar β_q is defined as

$$\beta_q = \frac{|\nabla f(\boldsymbol{X}_q)|^2}{|\nabla f(\boldsymbol{X}_{q-1})|^2} \tag{6.37}$$

Conceptually this method is similar to Powell's method, except that here each search direction is conjugate. Unlike the steepest descent method, here the information gathered (β) in previous iterations is used to determine the direction of search for the current iteration. Theoretically, this method will minimize a quadratic function of N variables in N or fewer iterations. In practice, it is necessary to restart the optimization process after every few iterations due to the non-quadratic nature of the problems, or due to numerical imprecision. In order to determine when to restart the optimization process, two criteria are used. First, when the one-dimensional search fails to improve the optimum, the process is restarted. Second when the slope in a search direction $\boldsymbol{s}$ with parameter α is positive, i.e.

$$df(\alpha)/d\alpha \equiv \nabla f(X_q)\cdot \boldsymbol{s}_q \geq 0, \tag{6.38}$$

numerical ill conditioning is indicated and $\boldsymbol{s}_q$ is set to $-\nabla f(X_q)$. The conjugate directions algorithm is described in detail in Procedure *Optimize_ND_CG*.

Procedure 6.5: *Optimize_ND_CG* (function $f()$, N, J, K, X^o, f^o, X^*, f^*, R, V_i, V, A, λ)
{
Purpose:

Perform unconstrained optimization of a function $f(X)$ (X being an N dimensional vector) using Fletcher and Reeves' conjugate gradients directions method.

Input:

Function to be optimized (augmented Lagrange function), $f()$;
Number of decision variables, N;

Number of equality constraints, J;
Number of inequality constraints, K;
Initial values for decision variable vector, X^o;
Value of the function at X^o, f_o.

Output:

Optimum values for decision variable vector X^*;
Optimum value of the function (augmented Lagrange function), f^*;
Optimum value of cost function, R;
List of constraint violations at the beginning of life at X_o, V_i;
List of constraint violations at the end of life at X^*, V;
Lagrange multipliers, λ.

Steps:

1. Evaluate the gradient of function $f()$ using the procedure

   Gradient (function $f()$, N, X, f, ∇f)

   Evaluate $a = -\nabla f \cdot \nabla f$

2. Evaluate the search-direction

   $s = -\nabla f.$

3. Find optimum point (α^*, X_* and f^*) in the search direction by 1-dimensional golden sections search method using the procedure

   Optimize_1D ($f()$, *Optimize_ND*, N, X, f, s, α_{max}, α^*, X^*, f^*, R, V_i, V)

4. $f = f^* = f_p^* = f_o$;

5. If $\alpha^* < \varepsilon$, exit.

6. Evaluate the gradient of function $f()$ using the procedure

   Gradient (function $f()$, N, X, f, ∇f);

   Evaluate $b = -\nabla f \cdot \nabla f$, and $\beta = b/a$;
   Evaluate the new search direction as, $s = -\nabla f + \beta s$;
   Assign $a = b$;
   Evaluate the slope of the objective function as, slope = $s \cdot \nabla f$.

7. If slope ≤ 0, go to step 2.

8. Find the optimum point (α^*, X_* and f^*) in the search direction by 1-dimensional golden sections search method using the procedure

Optimize_1D $(f(), Optimize\_ND, N, X, f, s, \alpha_{max}, \alpha^*, X^*, f^*, R, V_p, V)$

9. Check if the optimum solution has converged, using

$$|(f^*-f)|/f < \varepsilon, \text{ or, } |\Delta X| / |X| < \varepsilon, \text{ or, } \alpha^* < \varepsilon$$

where, ε is a small number.
Exit, if solution has converged, else goto step 6.

}

6.4.2 Variable Metric Method

The variable metric methods take into account the information gathered from the previous iterations, but now, instead of carrying that information in a single parameter (β), the information is stored in N parameters. The basic concept is to create an array which approximates the inverse of the Hessian matrix for quadratic functions as the optimization progresses. This method is considered more reliable and efficient compared to conjugate directions method. Here the search direction at iteration q is defined as

$$s_q = -\boldsymbol{H}\,\nabla f(X) \tag{6.39}$$

In the first iteration, the $N \times N$ matrix $\boldsymbol{H}$ is taken to be identity matrix $\boldsymbol{I}$, hence the search direction s_q is simply the steepest descent direction. At the end of iteration q, a new $\boldsymbol{H}$ matrix is defined as

$$\boldsymbol{H}_{q+1} = \boldsymbol{H}_q + \boldsymbol{D}_q \tag{6.40}$$

where D is a symmetric update matrix

$$D = \delta p p^T + [H y p^T + p(H y)^T]/\sigma. \tag{6.41}$$

Vectors p and y are defined as follows

$$p = X_q - X_{q-1}$$

$$y = \nabla f(X_q) - \nabla f(X_{q-1})$$

and scalars σ, τ and δ are defined as follows

$$\sigma = p.y$$

$$\tau = y^T H y$$

$$\delta = (\sigma+\tau)/\sigma^2$$

The variable metric method algorithm is described in detail in Procedure 6.6.

Procedure 6.6: *Optimize_ND_VM* (function $f()$, N, J, K, X^o, f^o, X^*, f^*, R, V_i, V, A, λ)
{
Purpose:

Perform unconstrained optimization of a function $f(X)$ (X being an N dimensional vector) using a variable metric method, or, Broydon-Fletcher-Goldfarb-Shanno (BFGS) method.

Input and Output:

Same as in the Procedure *Optimize_ND_CG*.

Initializations:

$f = f^* = f_p^* = f_o$;

Steps:

1. Initialize the pseudo-Hessian matrix $H[N \times N]$ to a identity matrix. Evaluate the gradient of function $f()$ using the procedure

   Gradient (function $f()$, N, X, f, ∇f)

2. Evaluate the search-direction,
   $s = -H\,\nabla f$.

3. Find optimum point in the search direction using 1-dimensional golden sections search method using the procedure

   Optimize_1D ($f()$, *Optimize_ND*, N, $\boldsymbol{X}$, f, $\boldsymbol{s}$, α_{max}, α^*, $\boldsymbol{X}^*$, f^*, R, V_p, V)

4. $\Delta \mathbf{X} = \mathbf{X}^* - \mathbf{X}$

5. Check if the optimum solution has converged, using

   $|(f^*-f)/f| < \varepsilon$; or, $|\Delta X| / |X| < \varepsilon$

   where, ε is a small number. Exit, if solution has converged.

6. Find the new search-direction $\boldsymbol{s}$ using

   Gradient (function $f()$, N, $\boldsymbol{X}^*$, f^*, ∇f^*)
   $y = \nabla f^* - \nabla f$
   $\sigma = \boldsymbol{p}.\boldsymbol{y}$
   $\tau = \boldsymbol{y}^T \boldsymbol{H} \boldsymbol{y}$
   $\beta = (\sigma+\tau)/\sigma^2$

   Evaluate the symmetric update matrix $\boldsymbol{D}[N \times N]$ as follows

   $$\boldsymbol{D} = \beta \boldsymbol{p}\boldsymbol{p}^T + [\boldsymbol{H}\boldsymbol{y}\boldsymbol{p}^T + \mathrm{p}(\boldsymbol{H}\boldsymbol{y})^T]/\sigma$$

   Evaluate the pseudo-Hessian matrix H as follows

   $$\boldsymbol{H} = \boldsymbol{H} + \boldsymbol{D}$$

7. Evaluate the slope in the search-direction, $\delta = s.\nabla f$. If $\delta > 0$, restart the optimization from step 1.

8. Update the decision variables vector and the gradient vector

$$X = X^*_p = X^*$$
$$\nabla f = \nabla f^*$$

9. Goto step 2.

}

6.5 Heuristic Based Optimization Approach

This section deals with the development of the integrated knowledge based system for the design of hydrodynamic journal bearings. The design process is quasi-procedural, wherein heuristic suggested design is validated using algorithmic procedures. The problem considered here assumes an a priori knowledge of the operating specifications, with the aim of generating an optimum bearing configuration. The task involves the preliminary synthesis of a suitable bearing configuration, dictated by a decision making capability and the user specified operating conditions. The preliminary configuration is analyzed to arrive at the optimum solution from a minimal manufacturing and life cycle operating costs standpoint. The procedure is iterative in nature, by which alternative configurations are synthesized using heuristic, and the best configuration is identified based on economic criteria (life cycle cost). A novel feature of the system is that the design constraint information is utilized as a feedback to the heuristic design segment of the program to improve the optimum design.

Knowledge based methods now provide a capability for representation of qualitative information, which can be coupled with traditional procedural analysis methods to arrive at practical design systems. Hitherto, such an approach was not

entirely feasible, due to the inherent difficulties in the representation and processing of valuable qualitative information that are an essential element of a successful design practice. Generic CAD systems available are usually limited to problem solving tasks in the algorithmic or deterministic domain. However, creative aspects in design are often ill-structured, and require the judgement of the designer. Therefore, attempts have been made in recent years to incorporate qualitative decision making capabilities based on experience in design procedures. It is generally accepted that the best approach is one which combines both heuristic and procedural methods. This section provides a design methodology which integrates knowledge based systems and algorithmic analysis methods for optimum life cycle design.

A schematic of the design system is shown in Figure 6.4. The system entails two distinct modules. The first is the heuristic decision making module, which incorporates an expert system CLIPS [CLIP88], and the second is an algorithmic module in 'C', which includes the procedural analysis and optimization.

6.5.1 Heuristic Module

The basic framework for the heuristic decision system is the CLIPS rule based environment. This utility invokes 'C' procedural programs, making possible convenient link between algorithmic procedures and heuristic. The system provides for the separation of the knowledge base, analysis procedures and domain independent inference engine. The procedure is initiated by the input of all the design requirements to the expert system. This system contains a knowledge base of pertinent design

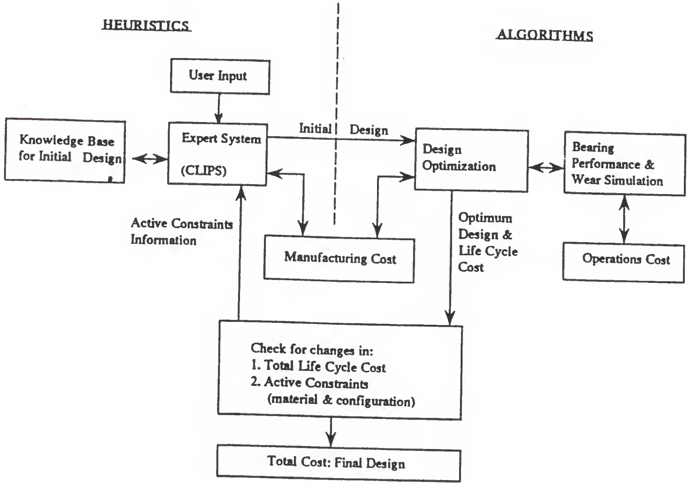


Figure 6.4: Design system combining heuristic and algorithmic approaches.

information, represented in the form of rules. These rules are written in the format required by the inference environment of the expert system (CLIPS).

6.5.1.1 Knowledge Representation and Execution. The syntax of a typical rule in the knowledge base is shown below.

```
(defrule <name> ["comments"]
(<first pattern>)
[(first pattern>)
....                    ;LHS
....
(<n th pattern>)]
=>
[(<first action>)
....                    ;RHS
....
(<nth action>)])
```

The example in Figure 6.5 illustrates a typical rule in CLIPS in the integrated design decision system. The LHS consists of one or more of the patterns which form the condition(s) to be satisfied, before a rule can be fired. An implicit 'and' is present in the LHS if more than one pattern is present. The RHS of the rule is a list of one or more action(s) to be performed as a result of the firing of the rule. The rule base also includes meta-rules. These are also treated as rules in the inference environment, each of which is a superset of two or more rules. The advantage of using meta rules is the efficiency of concatenating task specific rules. Furthermore, a better control over evaluations of rules is possible. In a basic execution cycle, the knowledge base is examined to see if the conditions of any rules are satisfied. This is done by simple matching of the data with the conditions. If the fields of the data asserted as variables,

match the conditions of a rule, then the rule is activated for firing. In the case when more than one rule match, all the matched rules are activated and pushed onto a stack. The most recently activated rule has the lowest priority and is placed at the bottom of the stack. The top rule is selected and executed; as a result, new rules may be activated and some previously activated rules may be deactivated. This cycle is recursive until no further pattern matches are possible.

```
(defrule rule5                                      ;name of the rule
?f1 <- (parameter ?p)                               ;parameter variable p
?f2 <- (diameter ?d)                                ;diameter variable d
=>
(if (< ?p 2000.)                                    ;for p less than 2000
        then
                (bind ?c (+ (* 0.00035 ?d) 0.00015))
                (assert (clearance ?c))             ;variable c is a fact
else
                (bind ?c (+ (* .0004 ?d) .0002))    ;c = 0.0002 + 0.0004 * d
                (assert (clearance ?c)))
(retract ?f1 ?f2))                                  ;remove facts f1 and f2
```

Figure 6.5: Knowledge representation in CLIPS - A sample rule.

6.5.1.2 Knowledge Base for Initial Design. The pertinent information was compiled from the references used in industrial design practice [Full84, Neal73, Tool83]. The information supplied to the expert system includes the bearing application domain, load, diameter of the shaft, and speed of operation. A partial description of the domain knowledge is included in Tables 6.1, 6.2 and 6.3. Table 6.1 lists different

bearing configurations and corresponding cost factors assigned to them. It also lists the factors for oil flow rates through those bearings. The plain journal bearings has been assigned a factor of 1.0 for both these factors. Table 6.2 lists different bearing materials and associated material cost factors, harnesses and machining cost factors. These cost factors are used to evaluate the costs in different stages of the life-cycle. Table 6.3 lists a range of bearing applications and associated materials, tolerances and L/D ratios used in the industry and as given in the handbooks. This data in these tables is used in the knowledge base for the selection of initial bearing design.

Table 6.1: Journal Bearing Configuration Data and Cost Factors.

Configuration	Configuration Cost Factor (α)	Relative Oil Flow (Q_r)
Axial Groove Square Edge	1.5	1.8
Axial Groove Round Edge	1.4	2.0
Circumferential Groove Square Edge	1.3	4.4
Circumferential Groove Round Edge	1.1	4.9
Plain	1.0	1.0

Table 6.2: Journal Bearing Material Data and Cost Factors.

Material	Material Cost Factor (β)	Hardness (H_B)	Machining Cost Factor (γ)
Lead-based Babbitt	1.0	21	1.0
Tin-based Babbitt	1.08	25	1.2
Cadmium Alloy	1.45	35	1.06
Copper-Lead	1.3	25	1.02
Bronze	0.9	60	1.16
Aluminum Alloy	1.2	45	1.1
Silver	3.0	35	1.02

Table 6.3: A Selected List of Bearing Applications, Materials, Tolerances, and L/D Ratios.

Application Area	Bearing Material	Tolerance (in)	L/D
Automobile engines	Lead based babbitt, copper-lead	0.0001-0.0007	0.75-1.75
Diesel engines	Lead-based babbitt, copper-lead	0.0001-0.0007	0.75-2.0
Air compressors	Lead based babbitt	0.0002-0.0004	1.0-2.0
Railway cars	Cadmium	0.0005-0.0009	1.8-2.0
Steam turbines	Tin based babbitt	0.0001-0.0003	1.0-2.0
Generators & motors	Lead based babbitt	0.0009-0.0015	1.0-2.5
Machine tools	Lead based babbitt	0..0001-0.0004	1.5-4.0

The task assigned to the expert system includes the identification of the initial design parameters. Specifically, they are the bearing material, configuration (type), length of the bearing, clearance between shaft and bearing, and lubricant viscosity. The selection of the material for the bearing is chosen based on the application domain of the bearing. The chosen material has a specified range of hardness, which defines the material wear factor. The application is categorized based on the type of industrial application. This categorization helps in the generation of initial values for clearance (C), length (L), and viscosity of the lubricant (μ), to be used in the optimization phase. A range of allowable clearances, lengths and viscosities is available for each application area. A simple approach would be to take the average values of the different ranges as an initial design. The initial bearing configuration and relative oil flow (Q_r) are

selected for the given application in accordance to tables given in Appendix A. The configuration controls the relative oil flow, which is an important component in performance evaluation. For example, axial or circumferential grooves and holes when introduced in the standard plain bearing have a significant effect on the oil flow. The next task is the identification of the initial tolerance on the bearing diameter. The tolerance significantly affects the selection of the manufacturing process and bearing performance and hence, the total cost of the bearing. The tolerance selection is based on the application domain and the material selected previously. All the selected parameters were introduced into the optimizer to achieve final design.

6.5.2 Algorithmic Module of Heuristic Based System

The design specifications which are selected by employing heuristic, are utilized in this module to attain an optimum design. The process involves minimization of the objective function, subject to the design constraints. Frequent references are made to the bearing simulation module, to evaluate the bearing performance and operating cost. It also makes references to the heuristic module for evaluating the manufacturing cost.

Optimization. The constrained minimization problem is converted to an unconstrained problem, using augmented Lagrange multiplier method presented in section sections 6.3 and 6.4. This is solved employing a conjugate directions algorithm, with one dimensional search performed using golden sections. The objective function evaluation is performed by invoking bearing simulation module, and by reference to the heuristic.

Bearing performance evaluation. Hydrodynamic and thermo-hydrodynamic models of lubrications are utilized to evaluate the operating characteristics of the journal bearing such as maximum pressure (P_{max}), temperature rise (ΔT), oil flow (Q), minimum film thickness (h_o), and coefficient of friction (f). The analytical relationships describing the journal bearing performance and constraints are given in presented in Chapter 2.

Operating cost evaluation. In order to evaluate the operating cost, the bearing performance is simulated as a function of time as described in Chapter 3. The underlying assumption is that as the bearing material wears, the clearance correspondingly increases with time. The time history of the performance parameters and total cost is continuously monitored and the optimum condition determined when the life cycle cost/hour reaches a minimum. That condition provides the best solution which can be obtained from the initial design conditions. In other words, it represents the local optimum solution resulting from the output of the heuristic segment of the system.

The bearing simulation is performed by an incremental increase (ΔC) in the clearance due to wear. The components of the operating cost can be formulated as sum of cost due to energy loss and cost of oil replacements. As the bearing wear progresses, there is a deterioration in its performance. Due to this, one or more of the constraints may be violated. One of the possible ways to rectify the problem is by replacement of the oil in the bearing which has a cost associated with it. If by replacing the oil, a successful constraint satisfaction can be achieved, then the

simulation is continued. Otherwise, the bearing is presumed to have reached the end of its useful physical life. The other criteria to end a bearing simulation is based on its economic life. This occurs when the total life cycle cost per hour of operation reaches a minimum.

6.5.3 Feedback Feature

A feedback feature is also provided in the heuristic based system, which facilitates the verification of changes in active constraints and total life cycle cost. The objective of this feature is to provide, when possible, alternative designs, which may lead to an improved optimum design. There are two main parameters for which alternatives are provided by the expert system, when certain constraints are active or violated. These parameters are bearing material and configuration. The heuristic module receives constraint information from the algorithmic module, and suggests possible changes. If for example, the temperature constraint is active, one solution is to achieve a higher oil flow, to curtail the rise in temperature. For such an action, heuristic would consider a different configuration or a different bearing material, which can withstand higher allowable temperatures. Out of the two possibilities, heuristic would suggest the solution with a relatively lower manufacturing cost factor. The modified set of parameters are passed on to the algorithmic module and the whole process is repeated until the life cycle cost for the alternative designs shows no further improvement.

6.6 Implementation of Integrated Decision System

The dynamic programming approach presented in the last chapter, the multilevel optimization and heuristic approaches presented in this chapter were implemented. The algorithmic programs were written in C++ language because of its flexibility in designing the data structure and the programs. Approximately 6000 lines of programs were written. All the relevant programs for dynamic programming, constrained and unconstrained optimization were developed, and no commercial package was used.

The developed programs are modular and designed to be very general and flexible. For example in the dynamic programming module, the number of stages, decision variables at each stage, number of descretizations at each stage and the number of constraints are changeable. When an objective function or constraint need to be changed, one has to only change the relevant modules which evaluate them. The program modules which evaluate the objective function or constraints, are separate from the program modules which optimize these functions. Hence when the optimization method need to be changed, one only has to modify the relevant modules. The number of decision variables, constraints and objective functions are not hard coded and can be easily changed.

The most exacting and time consuming part of the implementation was the development of the module for simulation of the journal bearings. The development was achieved after a repeated process of trial and error — where many assumptions about the physics of the bearings were made and the results verified by running the

simulations. Many times, the assumptions regarding the physical behavior and deterioration of the bearing performance were modified as a result of simulations so that finally the bearing life simulations now bear a resemblance to the actual bearing performance through its life. Literally thousands of such iterations of assumption making were performed, leading to convergence into "sound assumptions", which are presented in this thesis.

6.7 Summary

The integrated design methodology presented in this chapter combines heuristic and algorithmic approaches in the integrated system for the optimum design of machine elements for minimum life cycle costs. The findings of the algorithmic module are used by the system in a feedback feature for enhancing the utility of heuristic. The system is capable of selecting the configuration which minimizes the cost of providing specified tasks, together with material specifications, design parameters, as well as manufacturing process and tolerances. Although a bearing design is considered for illustration, the methodology offers sufficient flexibility to be utilized in other design domains.

CHAPTER 7
DESIGN OPTIMIZATION EXAMPLES

This chapter presents a few examples of how the integrated decision system is used to optimize the life-cycle for journal bearings. In the first two examples the same bearing, manufacturing and quality parameters are used and the system performance is based on hydrodynamic and thermo-hydrodynamic lubrication models respectively. Using the integrated design decision system, the optimum bearing designs are obtained and the results are then compared. In order to compare the difference between the two analytical approaches, the journal bearings optimized using hydrodynamic theory are subsequently analyzed using thermo-hydrodynamic theory. An examples is also given for a design optimization using the combined heuristic and algorithmic procedures.

7.1 Example 1 — Hydrodynamic Model

In this example the behavior of the bearing is assumed to be governed by the hydrodynamic lubrication theory. The task is to design the bearing for optimum life-cycle cost when the bearing length L, initial bearing clearance C_i, lubricant oil viscosity μ and tolerance (Δ_D) are the design decision variables. The integrated design system is used with the multi-level optimization.

7.1.1 Design Specifications

The following bearing design, manufacturing and quality control parameters are specified.

Design Parameters

Bearing Diameter, D	1.75 inch
Bearing Speed, N	3000 rpm
Load, W_o	1000 lb
Bearing material	Lead-based Babbitt
Bearing material hardness, H_b	21
Journal material hardness, H_s	200
Contaminant hardness, H_c	100
Bearing wear coefficient, K_b	0.016
Journal wear coefficient, K_s	0.006
Rotor mass, M	20 lbm
Oil inlet temperature, T_i	110 °F

Manufacturing Parameters

Cost factors α, β, γ	1
Coefficients of tolerance equation	$k_0=0.2$, $k_1=0.5$, $a=1/3$
Coefficients of base cost	$k_2=25$, $k_3=1$

Quality Control Parameters

In-plant cost of rework/scrapping, A	$C_{M,base}$ \$
Cost/measurement of diameter, B	1.00 \$
Cost/process adjustment, C	5.0 \$
Average number of products between successive adjustments interval, u_o	100 units
Quality control limits, $\pm\delta_0$	0.0001 in
Process lag in units of products, τ	5 units

Operation and Support Parameters

Oil cost, r_o	0.04 \$/in^3
Energy cost rate, r_e	0.01 \$/kWhr
Total oil volume, V_o	250 in^3

Sequence of Design Iterations

Initial values for the bearing decision variables are assumed to be as follows.

Tolerance on bearing diameter, Δ_D	0.0004 in,
Bearing length, L	1.75 in,
Bearing clearance, C_i	0.001 in,
Lubricant oil viscosity, μ	1.0×10^{-6} Reyn,
Initial estimate for cost of external quality, C_{EQ}	0.0 \$.

The multi-level optimization algorithms presented in Figure 6.2 and 6.3 are used for optimization. The sequence of design iterations performed is described below and is also presented in Table 7.1.

7.1.2 Iteration 1

Once the value of input parameters has been specified, the internal quality cost C_{IQ} is evaluated. At level one optimization, the decision system optimizes the life-cycle cost rate (*LCC* divided by bearing life) with bearing length (L), initial bearing clearance at the beginning of life (C_i), and lubricant viscosity (μ) treated as decision variables. Life-cycle cost (*LCC*) is defined as

$$\begin{aligned} C_{LC} &= \text{manufacturing cost + operation cost} \\ &\quad + \text{internal quality cost + external quality cost} \\ &= C_M(L, D, \Delta_D) + C_O(L, D, C_i, \mu) + C_{IQ}(\Delta_D) + C_{EQ}(\Delta_D, C_O''(C_i)) \end{aligned} \tag{7.1}$$

Bearing life is obtained by simulating the bearing performance over time, and then choosing the minimum of its physical and economic life. An example of such a simulation and resulting cost components versus the operation time of the bearing are

showed in Figure 7.1. The optimization is carried out subject to constraints on temperature, maximum pressure, minimum film thickness, contaminant to oil ratio, bearing length to diameter ratio, bearing clearance, lubricant viscosity, minimum life of the bearing and dynamic stability of the bearing. These constraints can be described as follows:

minimum oil film thickness:

$$h_o \geq h_{omin} = 5.0\times10^{-5} \text{ in},$$

maximum outlet temperature:

$$T_o \leq T_{max} = 300\ ^{o}\text{F},$$

maximum pressure:

$$P_{max} \leq P_{max,u} = 30\times10^{3} \text{ psi},$$

lower limit on the viscosity of the oil used:

$$\mu \geq \mu_{\min} = 0.01\times10^{-6} \text{ Reyn},$$

upper limit on the viscosity of the oil used:

$$\mu \leq \mu_{\max} = 50.0\times10^{-6} \text{ Reyn},$$

upper limit on length to diameter ratio:

$$L/D \leq (L/D)_{max} = 1.0,$$

lower limit on length to diameter ratio:

$$L/D \geq (L/D)_{min} = 0.25,$$

and the criterion ensuring the dynamic stability of the bearing described by equation 2.8.

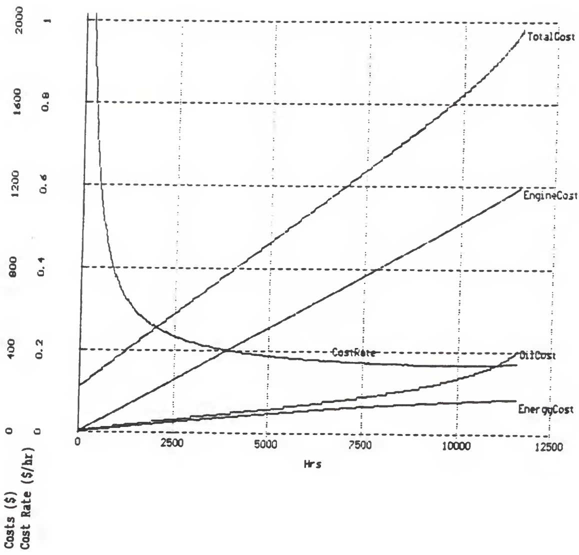


Figure 7.1: Life-cycle cost components versus time of operation obtained by bearing simulation.

Optimum values of the decision variables obtained after the first level optimization are L=1.67 in, C_i=0.0009 in, μ=2.151×10^{-7} Reyn. Other important results for the optimum design are bearing life=10800 hours, manufacturing cost C_M=208.8 \$, cost of energy loss $C_{O,e}$=303.25 \$, $C_{O,o}$=630 \$, and internal quality cost C_{EQ}=3.4 \$. Figure 7.2 displays the total cost and cost components as a function of initial clearance. The operation cost as a function of initial bearing clearance C_i around its optimum value (C_i^*=0.0009 in) is automatically obtained, and is used to obtain the 2nd derivative of operation cost with respect to initial clearance at its optimum value as follows

$$C_O''(C_i)|_{Ci=Ci^*} = 5.842\times10^{10}\ \$/\text{in}^2$$

Now all the four components of life cycle cost can be expressed in terms of tolerance Δ_D in the decision system as in equation (7.1). Figure 7.3 shows the life-cycle cost as a function of tolerance Δ_D. Thus the optimum value of tolerance Δ_D is obtained by using a directional search method. In the example, it produces tolerance

$$\Delta_D = 0.0001 \text{ inch},$$

external quality cost

$$C_{EQ} = 64.91\ \$,$$

total life-cycle cost

$$C_{LC} = 1206.96\ \$,$$

and cost rate

$$c_r = C_{LC}/\text{life} = 0.1117\ \$/\text{hour}.$$

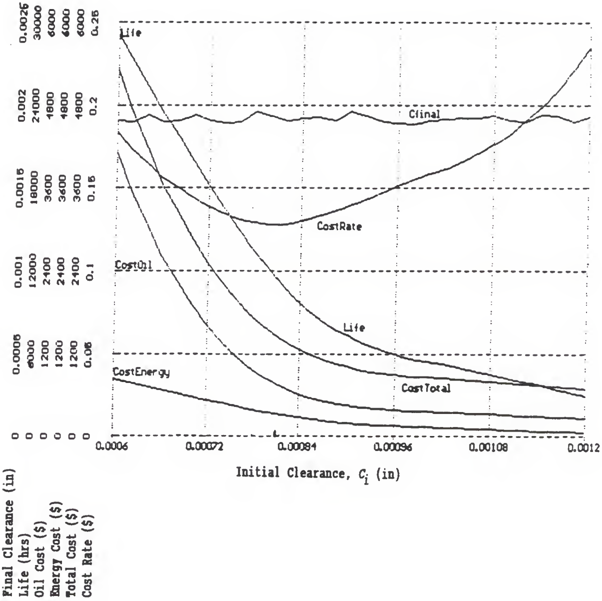


Figure 7.2: Life-cycle cost components versus initial clearance (C_i).

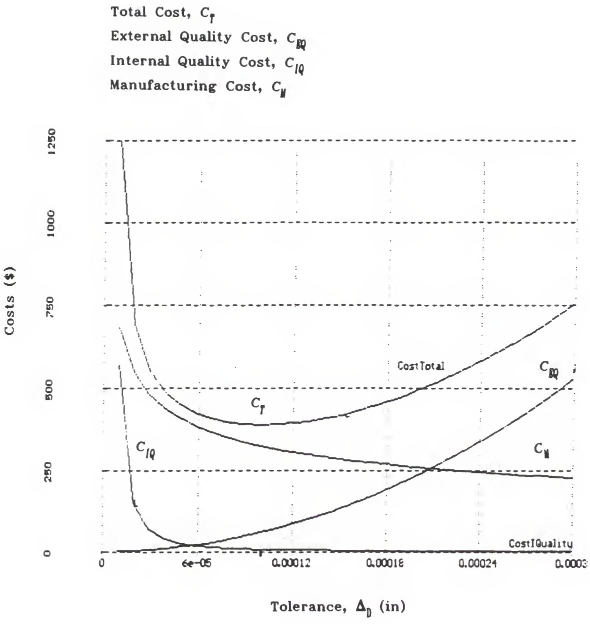


Figure 7.3: Life-cycle cost components versus bearing tolerance on diameter (Δ_D).

In this iteration, the first level optimization was carried out to find the optimum decision variables L, C and μ with the tolerance Δ_D treated as a fixed parameter. Then the second level optimization was done to find the optimum tolerance Δ_D with L, C, and μ treated as fixed. Iteration 1 is based on incomplete cost information, since the accurate values of internal and external quality costs is not available at this stage. The output of this iteration is used to compute the accurate costs in the 2nd iteration. Now, with these intermediate optimum values of the parameters and the tolerance, the 2nd iteration can be started.

7.1.3 Iteration 2

For iteration 2, the initial values of design variables as a result of iteration 1 are L=1.67 in, C_i=0.0009 in., $\mu=2.151\times10^{-7}$ Reyn, and Δ_D=0.0001 in. With these initial values, the first level optimization produces L=1.67 in, C_i=0.00081 in, $\mu= 2.016\times10^{-7}$ Reyn, life=11500 hours, C_M= 320.61 \$, energy cost $C_{O,e}$=319 \$, oil cost $C_{O,o}$=760 \$, C_{IQ}=30.71 \$. As in iteration 1, operation cost curve as a function of initial clearance C_i (a curve similar to that in Figure 7.2) is used to obtain the double derivative of operation cost as follows

$$C_O''(C_i^*) = 5.2295\times10^{10} \quad \text{at } C^*_i=0.00081 \text{ in}$$

$$\Delta_D = 0.0001 \text{ in}$$

As in iteration 1, a directional search method is used to obtain the optimum tolerance from costs versus Δ_D curve (a curve similar to that in Figure 7.3). It produces

$$C_{EQ} = 58.11,$$

C_{LC} = 1488.43,

Cost rate = $c_r = (C_{LC}/life) = 0.1294$ \$/hr.

7.1.4 Optimum Results

In each iteration, the decision system automatically goes through a similar cycle of the level-one and level-two optimization until the decision variables converge. Although, this example has been described using plots of various quantities, the decision system is completely automated and the optimization process does not require any reference to these plots. But the decision system can generate these plots if instructed by a user to do so. In this example, by the end of the 4th iteration, the decision variables and objective function converge, hence the computation process is stopped. Final values of important life-cycle parameters after the design has converged, are given in Table 7.1. The optimum values for quality control parameters obtained are as follows

Optimum process measurement interval, m	$3.64 \approx 4$ units
Optimum quality control limits, δ	0.000025 in
Predicted average process adjustment interval, u	$9.94 \approx 10$ units
Optimum process capability index, C_p	1.23

The plots of various physical quantities for the optimum bearing, as it is simulated over its life, are given in Figures 7.4 to 7.10. Only the first 260 hours of simulation is used here for plotting so that the graphs are clear and intelligible. When the oil is replaced in the bearing, most physical parameters (such as oil viscosity, pressure, temperature, contaminants, etc.) change suddenly. Thus the discontinuities

(kinks) in these simulation plots indicate the time when the oil is replaced in the bearing. For example, in Figures 7.4 - 7.10, the oil is replaced at 100 and 210 hours.

Figure 7.4 shows the plots of the Sommerfeld number (S) and the modified Sommerfeld number during the bearing life simulation. Figure 7.5 shows the plots of the wear factor (WF) and oil flow rate (Q) during simulation. The wear factor in the bearing is given by equation (3.11a) and it increases with time due to increased contaminants, oil temperature, and pressure until the time the oil is replaced. The bearing oil flow decreases with time.

Dimensionless rotor mass (drm=$MC\omega^2/W$) and stability criterion ($\psi(\sigma)$, a function of modified Sommerfeld number σ), which are the quantities used to determine the dynamic instabilities in the bearing, are plotted in Figure 7.6. With time, the bearing clearance (C) increases due to wear thus increasing the dimensionless rotor mass. For clarity, the two quantities are plotted on a different scale.

Figure 7.7 shows the plot of average viscosity (μ) and average temperature (T_{avg}) of the oil film in the bearing. Because of the contaminants and sludge generated over time, the viscosity rises until the oil is replaced. The temperature rise takes place because of the increased viscosity and reduced oil flow rate through the bearing. A plot of maximum pressure (P_{max}) in the oil film is shown in Figure 7.8. The maximum pressure decreases until the oil replacement, because the pressure distribution across the oil film becomes flatter. Figure 7.9 shows plots of minimum film thickness (h_o) and coefficient of friction (f). Until oil replacement, both the quantities rise because of the increasing oil viscosity. Figure 7.10 shows the plots of bearing clearance (C) and ratio

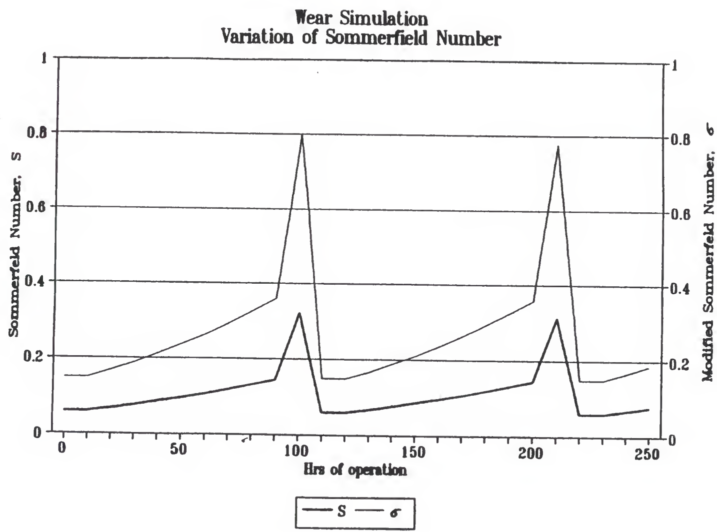


Figure 7.4: Sommerfeld number (S) versus hours of operation during simulation.

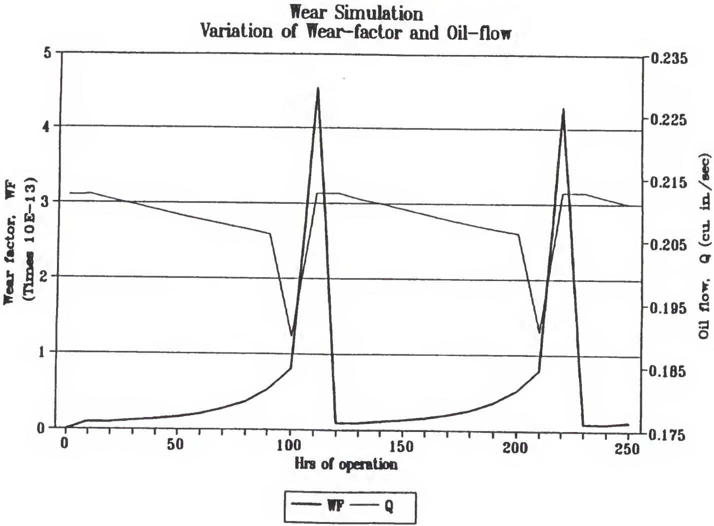


Figure 7.5: Wear factor (WF) and oil flow rate (Q) versus time of operation during simulation.

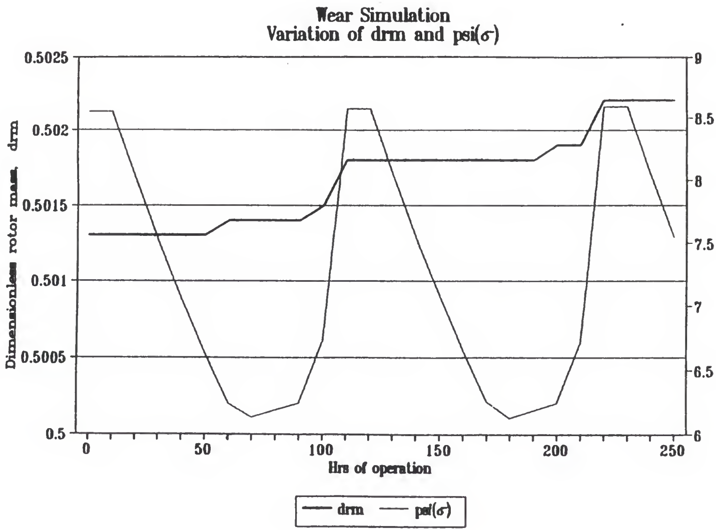


Figure 7.6: Dimensionless rotor mass (drm) and $\psi(\sigma)$ versus hours of operation during simulation.

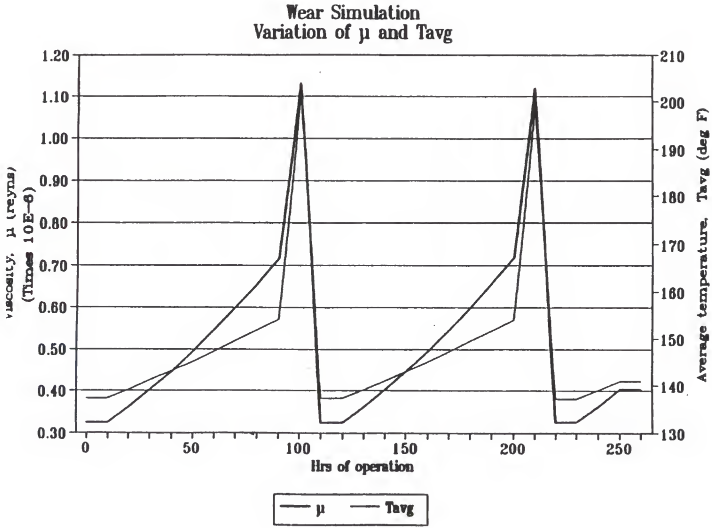


Figure 7.7: Oil viscosity (μ) and average temperature (T_{avg}) versus hours of operation during simulation.

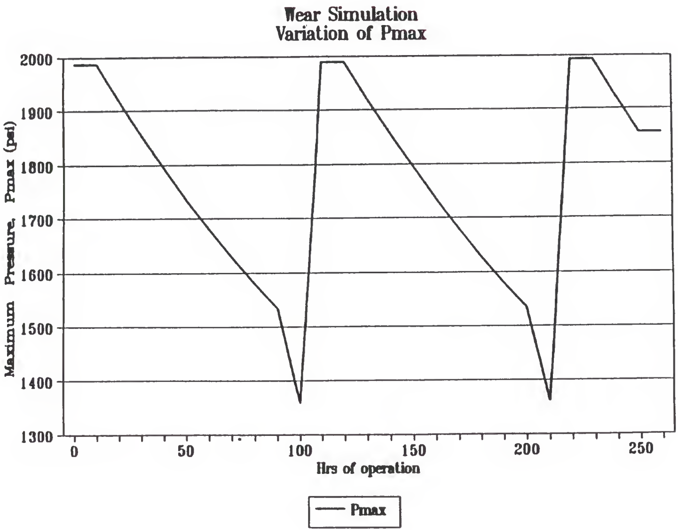


Figure 7.8: Maximum pressure (P_{max}) versus hours of operation during simulation.

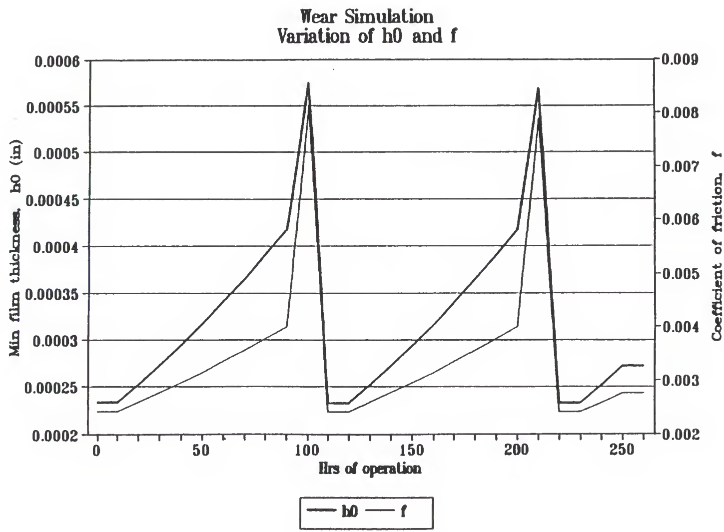


Figure 7.9: Minimum film thickness (h_o) and coefficient of friction (f) versus hours of operation during simulation.

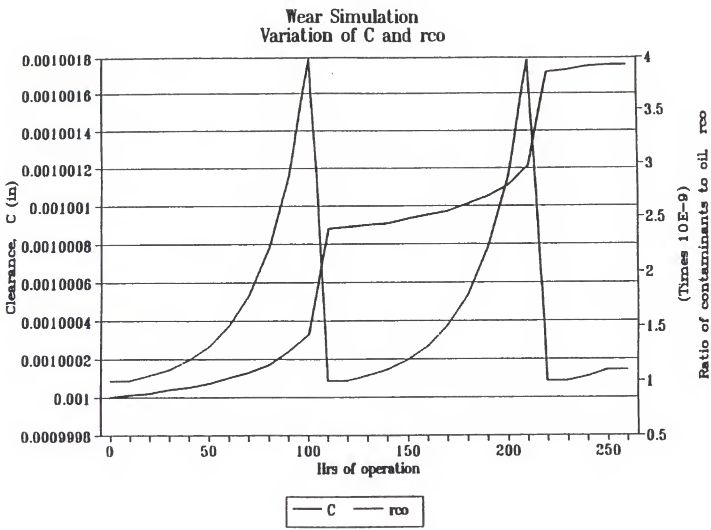


Figure 7.10: Clearance (C) and ratio of contaminants to oil (r_{co}) versus hours of operation during simulation.

of contaminants to oil (r_{co}). Due to bearing and journal wear, the clearance increases continuously and the contaminants increase until the oil is replaced.

7.1.5 Exploration of Design Space

Figure 7.1 is a plot of various cost simulations as the bearing goes through its life. As expected, The total cost (*LCC*) keeps rising over the life. More frequent oil changes at the end of bearing life increase the slope of oil cost. As expected, the cost-rate (*LCC* divided by the number of hours of operation) keeps decreasing. In this particular example, the simulation is terminated because the bearing has reached the end of its economic life where the cost-rate curve starts rising.

Figure 7.2 shows the plots of various costs, cost-rate, C_{final} (final bearing clearance at the end of life), and bearing life for optimum designs as a function of C_{initial} (bearing clearance at the beginning of life). Note that the final clearance has almost a constant value of 0.0019 inch where the bearing approaches the dynamic instabilities. Life keeps decreasing as the initial clearance increases since the increasing clearance reduces the allowance for bearing wear. Similarly, the oil cost and energy cost decrease with increasing C_{initial}. It is interesting to note that the cost-rate curve has a clear minimum at C_{initial}=.00081 approximately, hence it defines the optimum design clearance for the bearing.

Figure 7.3 plots various costs of optimum designs as a function of bearing tolerance on diameter Δ_D. Again, as expected, the external quality cost (C_{EQ}) increases with increasing tolerance, since the loose tolerance causes the bearing performance to

deviate from its optimal value. The cost of internal quality (C_{IQ}) drastically decreases with increasing tolerances. This is because the loose tolerances require less frequent quality control inspection, process adjustments and manufacturing cost. The quantity of interest here is the total cost (manufacturing cost + internal quality cost + external quality cost) which has a minimum at Δ_D=.0001 inch, confirming the hypothesis that there exists an optimum tolerance.

Table 7.1: Results of the sequence of iterations for parameter and manufacturing tolerance optimization for example 1.

Iterations / Parameters	Initial Values Chosen	Iteration 1 Results	Iteration 2 Results (optimum)
Length, L (in.)	1.75	1.67	1.67
Clearance, C_i (in.)	0.001	0.0009	0.00081
Oil Viscosity, μ (Reyn)	1.0×10^{-6}	2.15×10^{-7}	2.01×10^{-7}
Tolerance, Δ_D (in.)	0.0004	0.0001	0.0001
Mfg. Cost, C_M (\$)		208.80	320.61
Internal Quality Cost, C_{IQ} (\$)		3.40	30.71
External Quality Cost, C_{EQ} (\$)	0.00	64.91	58.11
Energy Cost, $C_{O,e}$ (\$)		303.25	319.00
Oil Cost, $C_{O,o}$ (\$)		630.00	760.00
$C_O''(C_i)$, (\$/in^2)	-	5.842×10^{10}	5.23×10^{10}
Life Cycle Cost, C_{LC} (\$)	-	1206.96	1488.43
Life (hours)	-	10800	11500
Cost-Rate, c_r (\$/hour)	-	0.1117	0.1294

7.2 Example 2 — Thermo-hydrodynamic Model

In this example, thermo-hydrodynamic theory is used to model the journal bearing with same design, manufacturing and quality control parameters as in example 1. The major differences between the hydrodynamic lubrication model and the thermo-hydrodynamic lubrication model are explained in Chapter 3.

In this section the design optimization is carried out for a bearing using two models. The optimum bearing design problem of example 1 and 2 is repeated here for three different loads (250, 1000, 2000 lb) and 4 different speeds (1000, 3000, 5000, 7000 rpm). Thus, twelve different combinations of loads and speeds are used as design specifications for design optimization using both hydrodynamic and thermo-hydrodynamic models. The result of design optimization for these two bearing models are then compared. Since we already know that the thermo-hydrodynamic model is more accurate, an interesting question is that how the optimum bearings designed with hydrodynamic models *actually perform* compared to the optimum bearings designed with thermo-hydrodynamic models. In other words, the objective here is to find the improvement in bearing performance and costs if the bearing is designed using the accurate design theory compared to the bearing designed with inaccurate design theory. This also yields the value (benefit) of right information/theory. Hence the bearings already optimized using hydrodynamic models are simulated using the thermo-hydrodynamic model. The comparison of these three cases now follows.

7.2.1 Optimum Cost-Rates

Table 7.2 and 7.3 show that the optimum cost-rate for both hydrodynamic and thermo-hydrodynamic bearings is very sensitive to bearing speeds and loads. With increasing bearing speed and load, higher pressures and temperatures are generated, resulting in increased energy loss and wear, thus increasing the cost-rates of the optimum designs. The bearings optimized using thermo-hydrodynamic model generally yield higher cost-rates.

The bearings designed with hydrodynamic model are also simulated as thermo-hydrodynamic bearings and the new cost-rates are given in Table 7.4. Comparing Tables 7.2 and 7.3 reveals that *actual* cost-rates in a bearing optimized using hydrodynamic theory are much higher than those predicted by hydrodynamic theory. These optimum cost rates as a function of bearing speed and load for both hydrodynamic and thermo-hydrodynamic model are plotted in Figure 7.11. Sometimes the bearings optimized using hydrodynamic model do not even *actually* satisfy the design constrains. When these bearings are simulated using the thermo-hydrodynamic model, their life turns out to be very short and hence cost-rates are very high. In Figure 7.12, the cost rates are compared for (a) bearings optimized using hydrodynamic model, and, (b) thermo-hydrodynamic simulation of bearings optimized using hydrodynamic model. The thermo-hydrodynamic simulation of optimum bearins sometimes results in very short life and consequently very high cost rates — this is evident in three cases (250 lb @ 7000 rpm), (2000 lb @ 1000 rpm), and (2000 lb @ 7000 rpm) in Figure 7.12. In Figure 7.13 the plot of normalized cost-rates as a function of bearing speed

is shown. Here the cost-rate is normalized using the cost-rate for bearing optimized using the thermo-hydrodynamic model, i.e.

$$\text{normalized cost-rate} = \frac{\text{actual cost-rate}}{\begin{array}{c}\text{cost-rate for the bearing optimized}\\ \text{using thermo-hydrodynamic model}\end{array}}$$

It is clear from Figures 7.12 and 7.13 that the actual cost-rates for the bearing optimized using hydrodynamic bearings are much higher, specially at higher speeds.

Table 7.2: Optimum Cost-Rates (10^{-2} \$/hr) — Bearings optimized using hydrodynamic lubrication model.

Load (lb) / Speed (rpm)	250	1000	2000
1000	1.44	3.73	4.50
3000	2.74	4.63	7.86
5000	4.64	6.92	10.39
7000	7.57	10.95	15.26

Table 7.3: Optimum Cost-Rates (10^{-2} \$/hr) — Bearings optimized using thermo-hydrodynamic lubrication model.

Load (lb) / Speed (rpm)	250	1000	2000
1000	1.96	4.09	7.01
3000	2.79	5.80	9.56
5000	4.95	7.79	11.00
7000	12.51	9.89	13.93

Table 7.4: Cost-Rates (10^{-2} \$/hr) — thermo-hydrodynamic simulation of bearing designed for optimum hydrodynamic model.

Load (lb) / Speed (rpm)	250	1000	2000
1000	1.60	6.30	∞
3000	3.30	10.00	14.3
5000	12.86	63.78	157.54
7000	∞	14.40	∞

7.2.2 Optimum Life

Table 7.5 shows that all the optimum hydrodynamic bearings achieve the designed maximum-life of 25000 hours. Except in one case (250 lb load, 7000 rpm speed), all the thermo-hydrodynamic bearings also achieve the same designed maximum-life of 25000 hours. When the bearings optimized using hydrodynamic lubrication theory are simulated as thermo-hydrodynamic bearings (Table 7.7), in most cases the life of the bearings turns out to be much shorter. A life of zero in Table 7.7 indicates that the initial bearing design does not even satisfy some of the design constraints. Figure 7.14 shows the normalized life of bearings for the two models. It is clear from these figures, that specially at high speeds, the actual bearing life is much lower compared to what is predicted by optimum solution based on hydrodynamic theory.

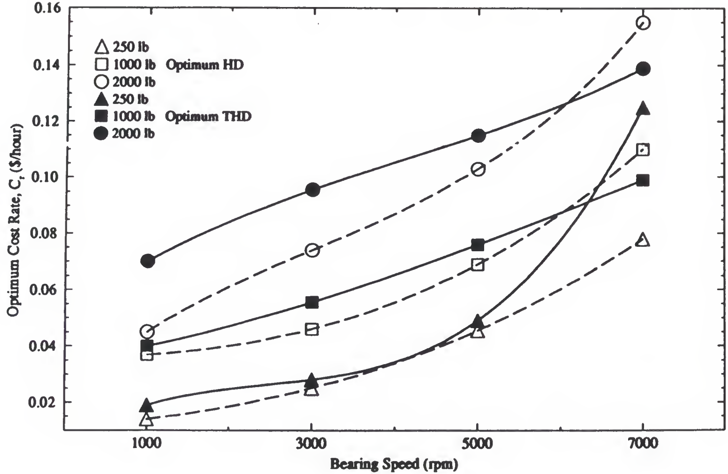
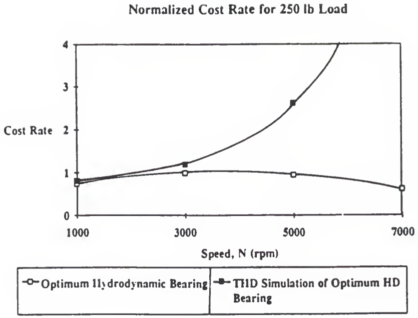


Figure 7.11: Comparison of cost rates for (a) bearing optimized using hydrodynamic model (optimum HD), and (b) HD bearings simulated using thermo-hydrodynamic model.



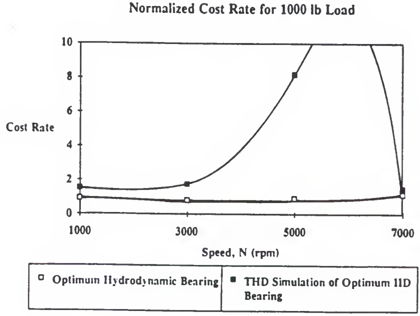


Figure 7.12: Normalized cost rates versus bearing speed for different bearing loads.

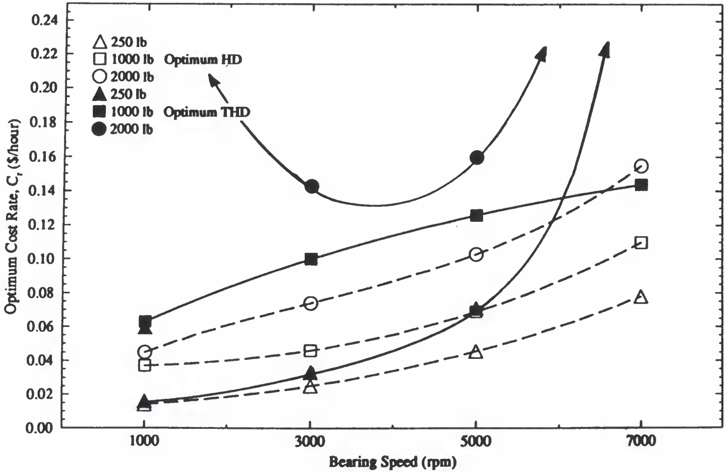


Figure 7.13: Cost rates versus bearing speeds at different loads for bearings optimized with hydrodynamic and thermo-hydrodynamic models.

7.2.3 Optimum Manufacturing Tolerances

Figure 7.15 shows the plots of optimum tolerances as a function of bearing speed (*N*) for three different bearing loads (250, 1000 and 3000 lb) for the two models. Table 7.8 and 7.9 list the values of optimum tolerances. Note that in both hydrodynamic and thermo-hydrodynamic models, the optimum bearing tolerances generally decrease with increasing bearing speeds. The higher bearing speeds require more precision, because at higher speeds, deviations from designed clearance affect the bearing pressures and temperatures more adversely than at lower speeds. The conditions of low bearing load and high speed (for example 250 lb, 7000 rpm) tend towards dynamic instabilities, as modeled by Lund and Saibel instability criterion. Hence low loads and high speeds require low bearing clearance (C_i) and tight tolerance. The conditions of high load and low speed (for example 2000 lb, 1000 rpm) also require tight tolerances. Bearings optimized using thermo-hydrodynamic model also requires tighter tolerances compared to the hydrodynamic model.

Table 7.5: Life (hrs) - optimum hydrodynamic lubrication.

Load (lb) / Speed (rpm)	250	1000	2000
1000	25000	25000	25000
3000	25000	25000	25000
5000	25000	25000	25000
7000	25000	25000	25000

Table 7.6: Life (hrs) - optimum thermo-hydrodynamic lubrication.

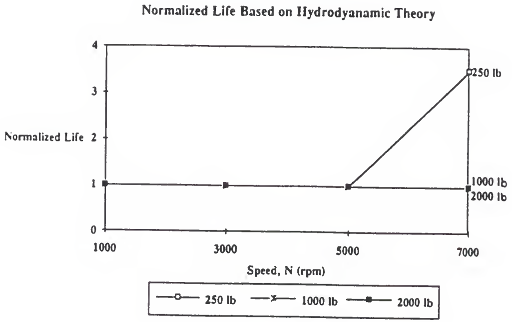
Load (lb) / Speed (rpm)	250	1000	2000
1000	25000	25000	25000
3000	25000	25000	25000
5000	25000	25000	25000
7000	7100	25000	25000

Table 7.7: Life (hrs) - thermo-hydrodynamic simulation of bearing designed for optimum hydrodynamic model. (7.3c)

Load (lb) / Speed (rpm)	250	1000	2000
1000	25000	16000	0
3000	25000	6500	6750
5000	3200	370	210
7000	0	5000	0

Table 7.8: Tolerance ($\pm$, $\times 10^{-3}$ inch) - optimum hydrodynamic lubrication.

Load (lb) / Speed (rpm)	250	1000	2000
1000	1.24	0.64	0.40
3000	0.46	0.65	0.46
5000	0.40	0.62	0.52
7000	0.30	0.43	0.50



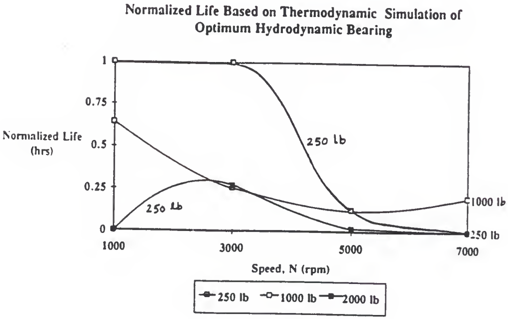


Figure 7.14: Normalized life versus bearing speed at different bearing loads.

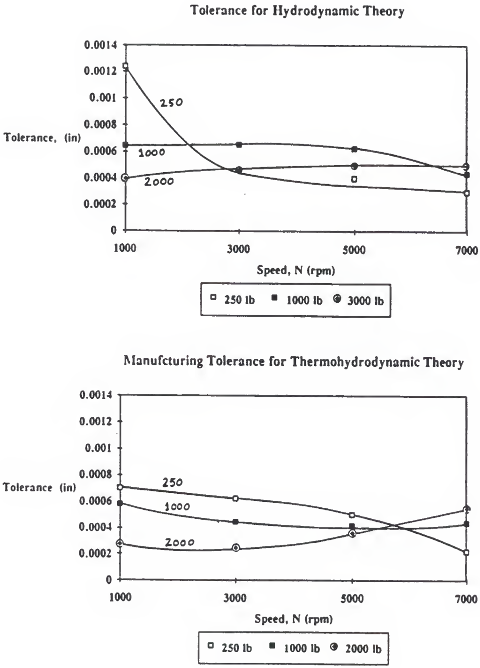


Figure 7.15: Optimum manufacturing tolerances versus bearing speed for different loads.

Table 7.9: Tolerance (±, $\times 10^{-3}$ inch) - optimum thermo-hydrodynamic lubrication.

Load (lb) / Speed (rpm)	250	1000	2000
1000	0.70	0.58	0.28
3000	0.62	0.44	0.25
5000	0.50	0.41	0.50
7000	0.22	0.43	0.55

7.2.4 Optimum Design Parameters

As a result of optimization for the 12 combinations of speeds and loads using both hydrodynamic and thermo-hydrodynamic models, the optimum values of design parameters were obtained. The optimum length to diameter ratio (*L/D*) for hydrodynamic and thermo-hydrodynamic models is plotted in Figures 7.16 and 7.17 respectively. The optimum initial clearance (C_i) for hydrodynamic and thermo-hydrodynamic models is plotted in Figures 7.18 and 7.19 respectively. The optimum relative oil viscosity (G_μ) for hydrodynamic and thermo-hydrodynamic models is plotted in Figures 7.20 and 7.21 respectively.

7.3 Heuristic Example

The heuristic based integrated design system described in section 6.5 was used for solving a sample design problem in this section. The task assigned is the design of a suitable bearing for the following specifications.

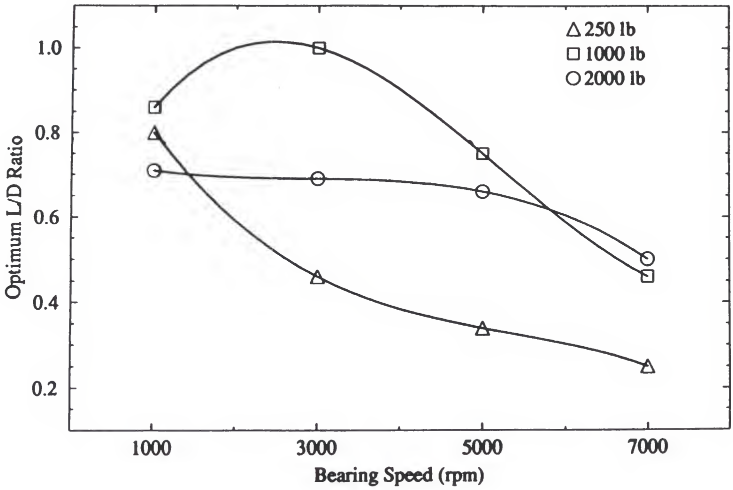


Figure 7.16: Optimum length to diameter (L/D) ratio vesus bearing speeds for different loads — hydrodynamic model.

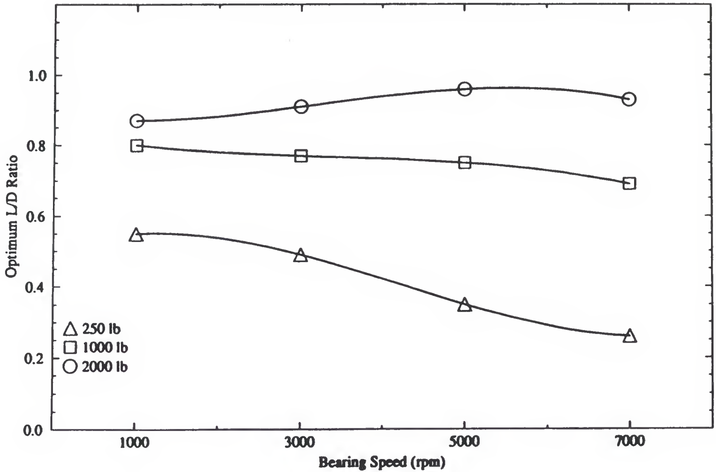


Figure 7.17: Optimum length to diameter (*L/D*) ratio versus speeds for different loads — thermo-hydrodynamic model.

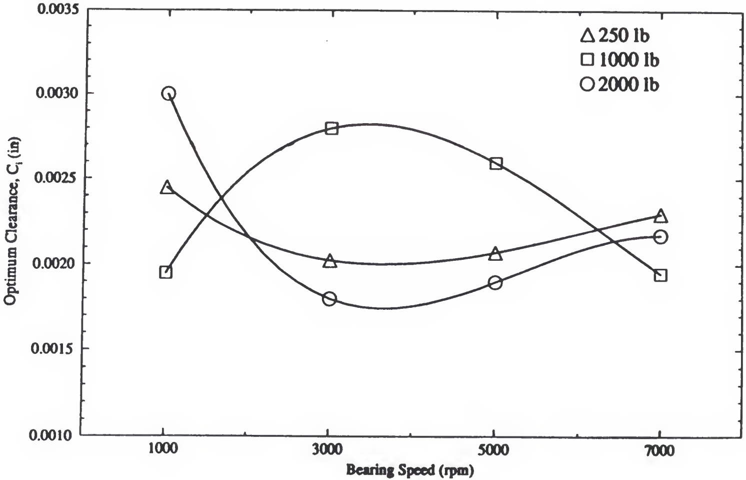


Figure 7.18: Optimum initial clearance (C_i) versus bearing speed for different loads — hydrodynamic model.

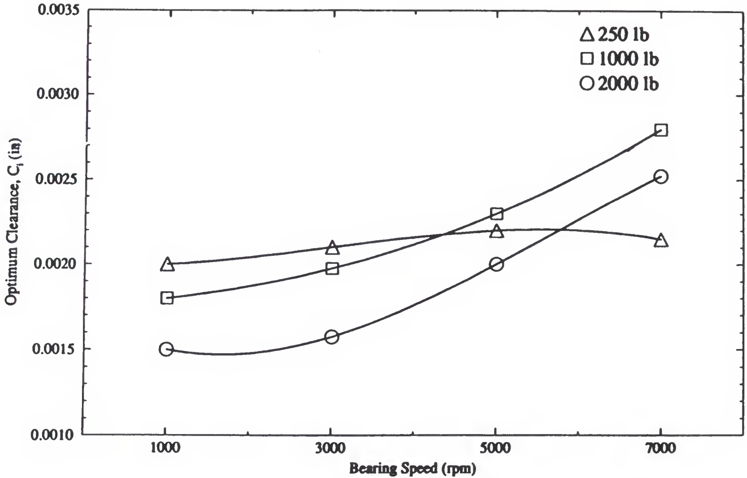


Figure 7.19: Optimum initial clearance (C_i) versus bearing speed for different loads — thermo-hydrodynamic model.

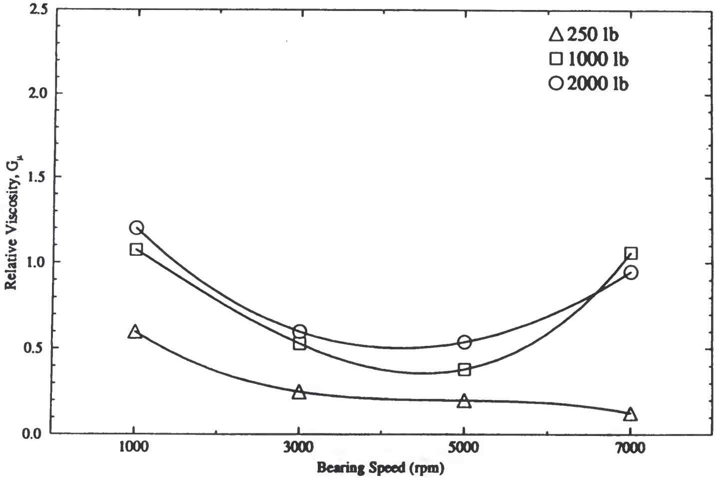


Figure 7.20: Optimum relative viscosity (G_μ) versus bearing speed for different loads — hydrodynamic model.

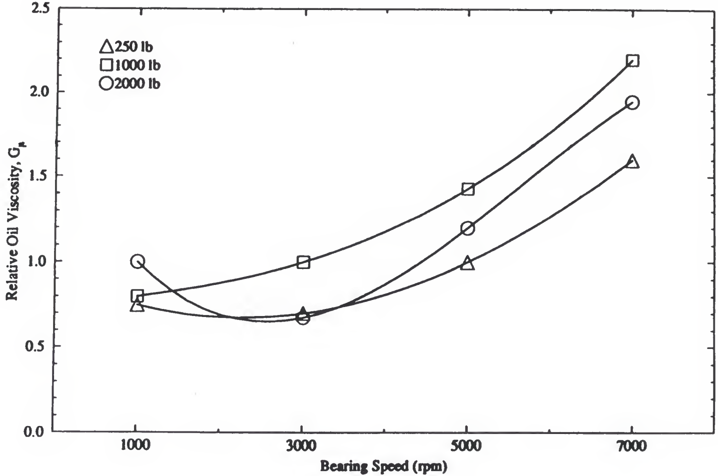


Figure 7.21: Optimum realtive viscosity (G_μ) versus bearing speed for different loads — thermo-hydrodynamic model.

7.3.1 Heuristic Module Input

Application	Compressor main bearing
Speed (rpm)	2000
Diameter (in.)	1.75
Load (lb)	1750

The output from the various modules are given below.

7.3.2 First Iteration Through Heuristic

The heuristic suggested the following initial design for the bearing.

Material	Lead-based Babbitt
Hardness	21
L/D Ratio	1.0
Clearance (in.)	0.001375
Tolerance (in.)	0.0003
Viscosity (reyn)	3.3×10^{-6}
Configuration	Plain bearing

Based on the initial design, the optimization module provided an optimum design with 11010 hours of life. At the end of life of the optimum bearing, the wear progresses to a point where minimum film thickness constraint and maximum pressure constraint are violated. The parameters for optimum design are as follows.

Life (hrs)	11010
Iterations	5
Length (in.)	1.5556
Clearance (in.)	0.001
Viscosity (Reyns)	0.6438×10^{-6}
Oil replacement interval	110 hrs
Manufacturing Cost ($)	200.03
Oil Cost ($)	543.75
Energy Cost ($)	175.32
Operations Cost ($)	719.07
Life Cycle cost ($)	919.10
Life Cycle Cost per hour of operation ($/hr)	0.0835

7.3.3 Second Iteration Through Heuristic

The active constraint information was passed from the algorithmic module to the heuristic module. Since, The bearing designed above has excessive wear; the heuristic suggested tin-based babbitt which is harder material (H_b=35).

Based on the modified design, the optimization module provided an optimum design with 14160 hours of life. As in the first iteration, the optimum bearing violates the minimum film thickness and maximum pressure constraints at the end of its life. The parameters for the second optimum design are as follows.

Life (hrs)	14160
Iterations	5
Length (in.)	1.5556
Clearance (in.)	0.001
Oil Viscosity (reyns)	0.6438×10^{-6}
Oil replacement interval	110 (hrs)
Manufacturing Cost ($)	220.32
Oil Cost ($)	693.75
Energy Cost ($)	225.41
Operations Cost ($)	919.16
Life Cycle cost ($)	1139.48
Life Cycle Cost per hour of operation ($/hr)	0.0805

7.3.4 Third Iteration Through Heuristic

In order to further reduce the wear rate, heuristic suggested bronze which is the hardest material (H_b=60) in the knowledge base. Interestingly, this time the algorithmic module produced an optimum design with more than 50000 hours of life. Up to the end of simulation, no constraint was violated. Since this design has the minimum life

cycle cost per hour of operation, with no active constraint, this is declared as the final design. Parameters for the final design are as follows.

Life (hrs)	50000
Active Constraints	none
Iterations	8
Length (in.)	1.5556
Clearance (in.)	0.001
Viscosity (reyns)	0.6438×10^{-6}
Oil replacement interval	110 (hrs)
Manufacturing Cost ($)	290.04
Oil Cost ($)	2449.60
Energy Cost ($)	795.70
Operations Cost ($)	3254.30
Life Cycle cost ($)	3535.34
Life Cycle Cost per hour of operation ($/hr),	0.0707

It is interesting to note that the feedback feature into the heuristic module modified the input to the algorithmic module and consequently produced a final design which is unconstrained, and has a lower life cycle cost per hour than the previously obtained local optimum. The material selected due to first feedback is tin based babbitt, which has a 20% higher hardness and is 10% more expensive in comparison with lead based babbitt. The material selected due to second feedback is bronze, which has a 114% higher hardness and is 25% more expensive in comparison with lead based babbitt. Hence finally bronze is selected since it gives the design with the minimum life cycle cost per hour of operation and with no active constraints.

Optimum Results

The output from the two modules during the design process are given in Table 7.10. It is interesting to note that the feedback feature into the heuristic module modified the input to the algorithmic module and consequently produced a final design which is unconstrained, and has a lower life cycle cost per hour than the previously obtained local optimum. The material selected due to first feedback is tin based babbitt, which has a 20% higher hardness and is 10% more expensive in comparison with lead based babbitt. The material selected due to second feedback is bronze, which has a 114% higher hardness and is 25% more expensive in comparison with lead based babbitt. Hence finally bronze is selected since it gives the design with the minimum life cycle cost per hour of operation and with no active constraints.

Table 7.10: Design iterations for the heuristic based optimization.

Bearing Parameters	Iteration 1	Iteration 1	Iteration 3
<u>Heuristic Module Output</u>:			
Material	Lead-based babbitt	Tin-based babbitt	Bronze
Hardness (BHN)	21	35	60
L/D ratio	1.0	-	-
Clearance (in.)	0.001375	-	-
Tolerance (in.)	0.0003	-	-
Oil Viscosity (Reyn)	3.3×10^{-6}	-	-
Configuration	plain bearing	plain bearing	plain bearing
<u>Algorithmic Module Output</u>:			
Iterations for Optimization	5	7	8
Life (hrs)	11010	14160	>50000
Length (in.)	1.5556	1.5556	1.5556
Clearance (in.)	0.001	0.001	0.001
Viscosity (Reyn)	0.6438×10^{-6}	0.6438×10^{-6}	0.6438×10^{-6}
Oil Replacement Interval (hrs)	110	110	110
Manufacturing Cost ($)	200.03	220.32	290.04
Oil Cost ($)	543.75	693.75	2449.60
Energy Cost ($)	175.32	225.41	795.07
Operation Cost ($)	719.07	919.16	3254.30
Life Cycle Cost ($)	919.10	1139.48	3535.34
Life Cycle Cost per Hour of Operation ($/hr)	<u>0.0835</u>	<u>0.0805</u>	<u>0.0707</u>

CHAPTER 8
MODELS FOR COST/BENEFIT ANALYSIS OF R&D AND NEW TECHNOLOGIES

The benefit of engineering or scientific research are many, some of which are not measurable in purely economic terms. However one of the roles of research and development in engineering is to reduce the level of uncertainty in design, through the improvement and refinement of information needed for the design process. In this regard, the benefits of engineering research may be assessed in terms of the savings in design and life-cycle cost accruing directly from an investment in research. The effectiveness of research and new technology may be analyzed from the stand-point of a tradeoff between the savings in life-cycle cost versus the research and development investment required to achieve such savings.

In this chapter, the cost effectiveness of research in further improvement in life-cycle cost is examined for the journal bearing examples. To do this, investment in research and its subsequent benefits reflected in the improved cost-rate and product life are evaluated. The cost and benefit of developing the thermo-hydrodynamic lubrication model is estimated. It is shown that the investment in R&D can both increase the profits to the manufacturer and savings to the user, though it results in higher priced bearing. The analysis also results in new price for the bearing. A schematic diagram in Figure 8.1 illustrates this concept.

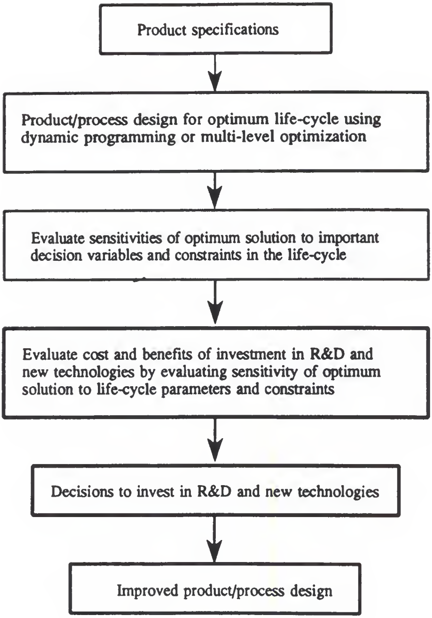


Figure 8.1: Sensitivity and cost-benefit analysis for lowering the life-cycle cost.

8.1 Cost/Benefit Analysis of Research

In this section an example illustrating the use of cost/benefit analysis for decisions in investment in research and development is presented. Suppose that the thermo-hydrodynamic model of lubrication did not exist, but it is known through experimentation that the hydrodynamic model of lubrication does not produce accurate results. It is also known that the deviation of experimental results from the predictions made by the hydrodynamic model is due to the thermal effects in the fluid film. Hence, a research effort has to be undertaken in order to develop the experimental setup and investigate the thermal effects in the fluid film. The cost and benefit evaluation of such an R&D effort for developing the thermo-hydrodynamic lubrication theory is presented in this section.

We will now compare the two models of lubrication for a bearing with speed N=5000 rpm and load W=1000 lb. The rest of the design, manufacturing, quality assurance, and operations and support parameter are taken to be same as in example 1 of the last chapter. Table 8.1 lists the important parameters for the optimum bearings for both hydrodynamic (HD) and thermo-hydrodynamic (THD) models.

From Table 8.1, it can be seen that the bearing designed using THD model has a life of L_{thd}=25000 hours, whereas the bearing designed using HD model actually has L_{hd}=6250 hours of life. Noting that the life of THD bearing is four times that of HD bearing, by the time an optimum THD bearing is replaced by the consumer, four HD

Table 8.1: Optimum HD and THD bearings for bearing speed N=5000 rpm, and load W=1000 lb. (HD=hydrodynamic, THD=thermo-hydrodynamic).

Cases / Parameters	Optimum HD Bearing	Optimum THD Bearing	THD Simulation of Optimum HD Bearing
Manufacturing Cost (\$), C_m	183.10	325.00	—
Internal Quality Cost (\$), C_{iq}	2.32	7.78	—
C_m+C_{iq}	185.42	332.78	
Oil Cost (\$), C_{oil}	565.00	685.00	35.00
Energy Cost (\$), C_e	975.19	844.05	3.97
External Quality Cost (\$), C_{EQ}	5.24	85.55	—
Life (hr)	25000	25000	6250
L/D	0.6948	.6868	—
Initial Clearance (in), C_i	.002594	.00228	—
Oil Viscosity (Reyn), μ_{avg}	$G\mu$=.31757	$G\mu$=1.4364	—
Tolerance (in), Δ_D	.001	0.0005	—
Cost-Rate (\$/hr)	.0692	0.07789	0.6378

bearings have been replaced. Hence the cost/benefit analysis is based on the longer of the two lives, i.e. 25000 hours. Assuming that the bearing is operated on average for h=4.28 hours per day, the lives of bearings in years are L_{thd}=16 year and L_{hd}=4 years. The analysis is based on the 1st year of sales of THD bearing, and 1st, 5th, 9th, and 13th year of sales of HD bearing.

8.2 R&D Cost

The cost of research and development ($C_{R\&D}$) can be estimated as follows.

Labor Cost		
1 theoretical engineer	100 \$/hr	
1 experimental engineer	100 \$/hr	
2 technicians	100 \$/hr	
Cost of 6 months of work	300 \$/hr	
	× 160 hr/month	
	× 6 months =	288,000 \$
Equipment cost estimate		300,000 \$
Total equipment and labor cost estimate		6×10^5 \$

8.3 Manufacturer's Profit

When the optimum HD bearings are produced, future worth at year y of manufacturer's profits in the first year at year y is given by

$$P_{hd1} = (p_{hd} - C_{mhd})n_{hd}\,(1+i)^y$$

Future worth at year y of manufacturer's cumulative profits over the time horizon of y years is given by

$$P_{hd,y} = (p_{hd} - C_{mhd})n_{hd}\,[(1+i)^y], \qquad \text{if } y \leq L_{hd}$$

$$P_{hd,y} = (p_{hd} - C_{mhd})n_{hd}\,[(1+i)^y + (1+i)^{y-Lhd}], \qquad \text{if } L_{hd} \leq y \leq 2L_{hd}$$

$$P_{hd,y} = (p_{hd} - C_{mhd})n_{hd}\ [(1+i)^{y} + (1+i)^{y-Lhd} + (1+i)^{y-2Lhd}], \qquad \text{if } 2L_{hd} \leq y \leq 3L_{hd}$$

$$P_{hd,y} = (p_{hd} - C_{mhd})n_{hd}\ [(1+i)^{y} + (1+i)^{y-Lhd} + (1+i)^{y-2Lhd} + (1+i)^{y-3Lhd}], \quad \text{if } 3L_{hd} \leq y \leq 4L_{hd}$$

Future worth of manufacturer's profits (due to 1st, 5th, 9th and 13th years of sales) at year L_{thd} of the profits over the time horizon of L_{thd} years is given by

$$P_{hd,Lthd} = (p_{hd} - C_{mhd})n_{hd}\ [(1+i)^{Lthd} + (1+i)^{Lthd-Lhd} + (1+i)^{Lthd-2Lhd} + (1+i)^{Lthd-3Lhd}]$$

Suppose that the manufacturer invests in research and development and the thermo-hydrodynamic model is developed. As a result, now the bearing is designed based on THD model, and the future worth at year y of the manufacturer's profits at year y is given by

$$P_{thd,y} = [-C_{R\&D} + (p_{thd} - C_{mthd})n_{thd}]\ (1+i)^{y}, \quad \text{for } 0 \leq y \leq L_{thd}.$$

where

p_{hd}	=	sale price of the bearing designed with HD model
C_{mhd}	=	manufacturing cost of the bearing designed with HD model
n_{hd}	=	Number of units sold per year for bearing designed with HD model
p_{thd}	=	sale price of the bearing designed with THD model
C_{mthd}	=	manufacturing cost of the bearing designed with THD model
n_{thd}	=	Number of units sold per year for bearing designed with THD model

i = annual interest rate

$(F/A,i,y)$ = Factor for future worth of an annuity of y years with yearly interest rate i and is given by

$$(F/A,\ i,\ y)=\frac{(1+i)^y\ -\ 1}{i}$$

$(P/A,i,y)$ = Factor for present worth of an annuity of y years with yearly interest rate i and is given by

$$(F/A,\ i,\ y)=\frac{(1+i)^y\ -\ 1}{i(i+i)^y}$$

Hence, change in manufacturer's profits in year y, when the bearing design basis is changed from HD to THD model, is given by

$$\Delta P_y = P_{thd,y} - P_{hd,y}$$

8.4 Consumer's Savings

When the bearing designed using HD model is used, the future worth at year y of initial cost to consumers can be given by

$C_{ihd,y} = (p_{hd} + p_i)n_{hd}\ [(1+i)^y]$, for $0 \leq y \leq L_{hd}$.

$C_{ihd,y} = (p_{hd} + p_i)n_{hd}\ [(1+i)^y + (1+i)^{y-Lhd}]$, for $L_{hd} \leq y \leq 2L_{hd}$

$C_{ihd,y} = (p_{hd} + p_i)n_{hd}\ [(1+i)^y + (1+i)^{y-Lhd} + (1+i)^{y-2Lhd}]$, for $2L_{hd} \leq y \leq 3L_{hd}$

$C_{ihd,y} = (p_{hd} + p_i)n_{hd} [(1+i)^y + (1+i)^{y-Lhd} + (1+i)^{y-2Lhd} + (1+i)^{y-3Lhd}]$, for $3L_{hd} \leq y \leq 4L_{hd}$

and future worth at year y of cumulative operation costs can be given by

$$C_{ohd,y} = C_{ohd}n_{hd} \times 365h \ (F/A,i,y)$$

Hence the total cost to consumers at year y, when HD model is used, can be given by

$$C_{hd,y} = C_{ihd,y} + C_{ohd,y}$$

When the thermo-hydrodynamic (THD) model is adopted as the design basis, the new initial costs to consumers are give by

$$C_{ithd,y} = (p_{thd} + p_i)n_{thd} \ [(1+i)^y], \ for \ 0 \leq y \leq L_{thd}$$

and the future worth at year y of operation cost can be given by

$$C_{othd,y} = C_{othd}n_{thd} \times 365h \ (F/A,i,y), \ for \ 0 \leq y \leq L_{thd}.$$

The total cost to consumers at year y, when HD model is used, can be given by

$$C_{thd} = (p_{thd} + p_i)n_{thd}(1+i)^y + C_{othd}n_{thd} \times 365h \ (F/A,i,y)$$

Hence the savings to consumers, by changing the design model from HD to THD, can be expressed as

$$\Delta S = C_{hd} - C_{thd},$$

where

p_i	=	Installation cost of the bearing at the consumers' site
C_{ohd}	=	Operation cost of HD bearing per hour of operation
C_{othd}	=	Operation cost of THD bearing per hour of operation

8.5 Price Determination

In these cost formulations the known quantities are $C_{R\&D}$, C_{mhd}, C_{mthd}. The unknown quantity is the price the manufacturer should charge for the bearing designed with THD model (p_{thd}). Three special cases are considered in order to determine the THD model price p_{thd}.

Case (a): Here it is assumed that over the time horizon of interest (L_{thd}), manufacturer's profit rise is equal to the savings to consumers. Here basically, the price is set in such a way that manufacturer passes half the profit increase to the consumers. This can be expressed as

$$\Delta P = \Delta S$$

Case (b): Here it is assumed that the manufacturer does not pass any profit increase to consumers. Thus the price is set such that

$$\Delta S = 0$$

Case (c): Here it is assumed that the manufacturer passes all the profit increase to consumers. Thus the price is set such that

$$\Delta P = 0$$

These three cases give three different equations, which are solved for three different values of the price of the THD bearing model. These prices along with the manufacturer's profit and consumers' savings are listed for two annual interest rates of 12% and 6% in Table 8.2 and 8.3 respectively.

The calculations show a steep rise in price of the bearing. For case (a) with the interest rates i=12% and 6%, the price of the bearing is determined to be $1171 and $1526 respectively. Even after a price rise, both manufacturer's profits and the consumers' savings increase. Thus the investment in R&D is justified.

Table 8.2: Manufacturer's profit increase and consumers savings for different prices of bearing designed with thermo-hydrodynamic model. Interest rate, i=12%.

Price of bearing designed with thermo-hydrodynamic bearing p_{thd}, \$	Increase in manufacturer's profit ΔP, \$	Increase in consumers' savings ΔS, \$
Case (a): 1171.36	1.89×10^9	1.89×10^9
Case (b): 1798.03	3.77×10^9	0.0
Case (c): 544.69	0.0	3.77×10^9

Table 8.3: Manufacturer's profit increase and consumers savings for different prices of bearing designed with thermo-hydrodynamic model. Interest rate, i=6%.

Price of bearing designed with thermo-hydrodynamic bearing p_{thd}, \$	Increase in manufacturer's profit ΔP, \$	Increase in consumers' savings ΔS, \$
Case (a): 1526.35	1.21×10^9	1.21×10^9
Case (b): 2478.81	2.42×10^9	0.0
Case (c): 573.89	0.0	2.42×10^9

CHAPTER 9
SUMMARY, CONCLUSIONS AND RECOMMENDATIONS

The successful development and implementation of the design methodology has brought into focus a variety of observations. This chapter starts with a summary of the research reported in this thesis. Some of the observations that became evident during the course of the research, and their implications on future research are then enumerated. Concluding remarks include a scrutiny of the modeling of different stages in the life-cycle, the solution strategies, the modifications needed to use the decision system for a different design domain, and experiences with the design decision system implementation.

9.1 Summary

In this section the research reported in this thesis is summarized. An investigation of the concept of optimum product design for life-cycle cost was undertaken. Most of the product life-cycle cost is committed at the design stage, hence, it is appropriate that the life-cycle considerations be included at the design stage itself. The idea is to investigate those parameters and constraints that effect the life-cycle cost of the product at the design stage itself.

The life-cycle was divided into four stages — 1. research, development and design, 2. manufacturing, 3. quality assurance, and, 4. operations and support. A journal bearing was chosen as the design domain because it is simple in configuration with few decision variables, yet presents all the difficulties, nonlinearities and constraints of a complex design problem. Its performance characteristics are also very sensitive to the decision variables.

The life-cycle cost was considered to consist of R&D cost, manufacturing cost, in-plant (internal) quality assurance cost, external quality cost (cost to consumers), and operations and support cost. Manufacturing cost was modeled using those design parameters that affect the cost (length, diameter, tolerance, material).

The operation and support cost was taken to be the cost of energy loss (due to friction) and the oil replacement cost through the life of the bearing. In order to evaluate the operations and support cost, the bearing and oil deteriorations are simulated through its life. The bearing life could be either its physical life, when it starts to violate any of the constraints, or its economic life, when the cost-rate (life-cycle cost divided by the number of hours of operation) curve reaches its minimum. Modeling of wear and deterioration of oil was one of the most sensitive part of the life-cycle cost.

Internal quality costs (in-plant) were modeled assuming that the process is being controlled by either an automatic system or a quality control chart such as $\overline{X}$ chart. External quality costs (to consumers) measure the cost of variability of performance from the optimum design. The measurement method of external quality costs in this research is represents an improvement over the previous methods published in literature.

For the life-cycle optimization, three different solution approaches were investigated and implemented. The first two approaches tried were algorithmic and the last approach combines algorithmic approach with the heuristic approach. The algorithmic optimization approaches implemented are very general in nature. The decision system developed is highly modular and could be easily modified for a different design domain.

The first optimization approach was to treat the life-cycle optimization as a dynamic programming problem. Dynamic programming approach is applicable and is very efficient when a problem can be thought of as consisting of serially connected black boxes (stages) with output of each stage being fed as input to the next stage. Each stage has an objective function, input parameters, output parameters, constraints and a set of decision variables. The objective is to choose all the decision variables subject to given constraints so as to optimize the total objective function, which is the sum of objective functions of all the stages. A very general dynamic programming based optimization system was developed where the number of stages and the number of decision variables at each stage can be varied.

A very general nonlinear constrained optimization module was developed which can handle a design vector of arbitrary size and any number of constraints. The constrained optimization problem is first converted into an unconstrained optimization problem by adding a combination of interior and exterior penalty functions to the objective function, and then by using augmented Lagrange multiplier method. The

unconstrained problem is optimized using conjugate directions or variable metric methods.

The second optimization strategy used was multi-level optimization approach, where the design optimization is decomposed into many levels. Here the design vector is decomposed into a global design vector and level-specific design vectors. In this research, the design vector was decomposed into two levels — first level being the design parameters and the second level being the tolerance. At each level of optimization, calls to nonlinear constrained optimization modules are made.

The third optimization approach tried was a combination of heuristic and algorithmic procedures. Ideally, the combination of the two procedures in a decision system can provide the best design tools. The heuristic system generates initial design based on the application domain of the bearing specifications supplied by the designer. The heuristic module can also process the feedback about constraint violations and modify the appropriate parameters so that the design remains within the feasible domain. The heuristic module is design domain specific and needs to be completely rewritten for a different design domain — but it can serve as a guideline for developing the heuristic module for a new design domain.

The decision system developed proved to be a useful tool in the optimum design in the domain of journal bearing. Many examples of parameter and tolerance design for a journal bearing for optimum life-cycle cost were presented. It is seen that a well defined optimum for parameters and tolerance exist. Values of optimum bearing length, clearance, viscosity, and tolerance obtained seem to be very reasonable.

The research illustrates the importance of incorporating manufacturing, quality assurance, operations and support, and cost/benefit analysis considerations at the design stage decisions itself. Using the decision system, the concept of identifying the need for research and development and justifying the investments in R&D were investigated. The cost/benefit analysis of alternatives in the life-cycle identifies the parameters and constraints that need to be improved. It also provides the justifications for investments in R&D and new technology.

9.2 Concluding Remarks

In this section, some of the observations that became evident during the course of the research are presented. Design for life-cycle is akin to the concept of *concurrent engineering* or *simultaneous engineering*. In very complex designs/projects, it may not be obvious that in order to reduce total costs, what improvements are necessary. Cost modeling, optimization and then sensitivity analysis can immensely help in determining where to invest the scarce resources. Cost/benefit analysis of alternatives available helps a decision maker.

Modeling of bearing wear and deterioration of oil was the most sensitive and time consuming part in the development of the decision system. An insight into the lubrication and tribology practice helps make a few intelligent assumptions, which can make the task of bearing life simulation manageable. The implemented decision system calculates the objective function using simulations of bearing life. Performing the life

simulations of the bearing was found to be very demanding on the computational resources. Each simulation requires an average of 10 minutes on a 80826 processor based PC with a 80287 math coprocessor. But it was necessary for accurate prediction of physical parameters and costs of the life-cycle. Because, thousands of such bearing simulations were to be conducted, a graphics plotting module was developed and was incorporated into the decision system, thus providing an instant access to the progress of the life-cycle simulation.

As a solution strategy, the dynamic programming was found to be computationally slower than the multi-level optimization approach. But in large problems with many more stages, dynamic programming could prove to be more efficient than multi-level optimization approach. The investigation also revealed that a combination of heuristic and algorithmic procedures in a decision system can provide the best design tool. One limitation of the heuristic based approach is that for each new design domain, the module must be rewritten.

The life-cycle optimization problem for a different design domain can be accommodated by readily modifying the algorithmic decision system. The algorithmic part of the decision system is modular in nature and can be modified. The dynamic programming module is also very general and it can incorporate an arbitrary number of stages and an arbitrary number of decision variables at each stage — subject to memory and storage capabilities of the computer.

In the multi-level optimization approach, the design domain optimization must be decomposed into many levels. Hence the designer has to decompose the design

vector into a global design vector and level-specific design vectors. Hence the designer has to incorporate this in the main program of the design module. He also has to make calls to optimization modules for each level of optimization. The nonlinear optimization modules require to be fed the normalized values of the design vector. A designer needs to replace the following modules in the algorithmic part of the system in order to use the decision system for a different design domain.

- Each design or life cycle constraints is represented by a subroutine in the system. Any number of constraints can be incorporated as a design domain requires.
- The input/output module provides the input/output venue for the parameter values and design vector.
- The journal bearing simulation module calculates the operation costs.
- The internal quality cost evaluation module can be replaced if a different quality control method is being used.
- The manufacturing cost evaluation module can also be changed to suit the problem under consideration.

9.3 Recommendations for Further Research

This section outlines the recommendations for further research and possible extensions to the design decision system which has been implemented. Better tribological models for wear are needed. Besides the improvements in modeling the

manufacturing cost, quality cost, reliability cost discussed in the following sections, additional factors or stages can be included in the life-cycle. For examples, cost of disposal of the replaced bearing could be incorporated.

9.3.1 Manufacturing Cost

Better cost models for manufacturing processes are needed. The manufacturing cost could include the following factors:

- The part parameters such as specified surface roughness, surface coatings etc.
- The part material characteristics such as machinability, material hardness, any heat treatment needed and material cost.
- Lot size of the parts.
- Process parameters (or machining parameters) such as cutting speed, depth of cut, feed rate, processing time, and tool material.
- Part handling parameters such as any special jigs and fixtures needed, setup time, and material handling time.
- The machine capacity including factors such as power rating and speed.
- The machine capability, the ability of the machine to attain design specifications, such as geometry, dimensional tolerances, and surface roughness.
- The machine availability, a factor varying between zero and one, which represents whether a machine is available for use when requested or it is highly in demand. This factor then can be used for applying a penalty for choosing a relatively unavailable machine.

Based on these manufacturing parameters and selected manufacturing process, one could calculate the fraction of the products (or parts) being produced which are defective. This information is very useful in evaluating the quality costs accurately.

One could include and make use of a plant-wise database on material costs and material availability. Similarly one could also make use of the databases on production machine capacity, capability and availability in order to make optimum process plans/scheduling for production. This data not only can be used for evaluating the costs, but also it can serve as constraints imposed by the manufacturing function to the designer. Better accounting practices can provide the data on costs associated with these factors. Already existing modules which produce and optimize the production schedules could be incorporated. Cost of research and development, innovations and investments in new manufacturing processes and machines could also be incorporated.

9.3.2 Quality Costs

Depending on the design domain, there can be different quality control plans used for example attributes or variable control charts. Integrated system could incorporate attribute control charts, and process control based on it.

9.3.3 Reliability Cost

Following is a brief presentation of the way reliability and maintainability consideration could be incorporated in life-cycle analysis. The product reliability and maintainability cost is the cost incurred due to scheduled preventive maintenance, cost

of failures and the cost of unscheduled maintenance following failures. It can be expressed as

$$C_R = N_f C_f + N_m C_m, \tag{9.1}$$

where N_f and N_m are, the expected number of failures (or, unscheduled corrective maintenance) and the number of replacements of unfailed parts (or, preventive maintenance) during the life-cycle of the product, respectively. C_m is the cost of replacing the enfilade parts including the loss due to product (or system) unavailability during preventive maintenance. C_f is the cost of parts failures and subsequent replacement during corrective maintenance, including the damage to other parts and the system. For the journal bearing example, "parts" may include any of the bearing parts or the lubricant oil. Total number of parts replacements is given by

$$N = N_f + N_m. \tag{9.2}$$

Assuming that the total life-cycle time is very long compared to the mean time between replacements (*MTBR*), we have

$$MTBR = t/N. \tag{9.3}$$

If $R(t)$ be the part reliability at time t, and assuming that the part is replaced after it has been in operation for time T, then *MTBR* can be expressed as

$$MTBR = \int_0^T R(t)\,dt. \tag{9.4}$$

Therefore from the above two equations, the number of parts replacements over a long time span t is

$$N = \frac{t}{MTBR} = \frac{t}{\int_0^T R(t)\,dt}. \tag{9.5}$$

Since only a fraction of the parts will survive to the next preventive maintenance, the number of unfailed parts replaced during the life-cycle of the product is given by

$$N_m = R(T)N = \frac{t\,R(T)}{\int_0^T R(t)\,dt}. \tag{9.6}$$

Moreover, since a fraction 1-$R(T)$ of the parts will fail, the number of failed parts replacement is given by

$$N_f = [1-R(T)]N = \frac{t[1-R(T)]}{\int_0^T R(t)\,dt}. \tag{9.7}$$

With expressions for N_f and N_m determined, total reliability cost C_R becomes a function of replacement time T. Now we can minimize the cost of reliability by making $dC_R/dT=0$, which can be reduced to

$$R(T) + \lambda(T)\int_0^T R(t)\,dt - 1 = \frac{C_m}{C_f - C_m} \tag{9.8}$$

which has to be solved numerically if the optimum T is to be estimated. Here $R(t)$ is the reliability function and $\lambda(t)$ is the failure rate defined as follows

$$\lambda(t) = -\frac{1}{R(t)}\frac{dR(t)}{dt} \tag{9.9}$$

If one assumes that the failure rate follows the widely used Weibull distribution, which can model a wide range of failure modes, we have reliability function as

$$R(t) = e^{-\left(\frac{t}{\theta}\right)^m}, \tag{9.10}$$

and failure rate given by

$$\lambda(t) = \frac{m}{\theta}\left[\frac{t}{\theta}\right]^{m-1} \tag{9.11}$$

Here θ is a scale parameter having dimension of time. Parameter m is a shape parameter which depends on the mode of failure. For failure rates increasing with time, $m>1$, and wear is one of the modes of failure. For failure rates constant in time, $m=1$, the mode of failure is random and the failure rate has an exponentially decreasing distribution. Assuming that failure cost C_f is much greater than the preventive maintenance cost C_m, and after performing a few algebraic manipulations, the optimum time of part replacement (MTBR) is given by [Lewi87]

$$MTBR^* \approx \theta\left[\frac{1}{m-1}\frac{C_m}{C_f}\right]. \tag{9.12}$$

Besides estimating the optimum time to replace parts, it is also important to find the optimum age of the whole journal bearing system. Many studies have been conducted to develop reliability based probabilistic failure models based on the wear rates of various mechanical components. The references [Skew85, Raze86, Raze87, Rhod88, Lamb88, Nels89] suggest the reliability based design methods for mechanical equipment such as clutches, brakes, seals, bearings, valves and gearboxes.

As an alternative to the deterministic models used to model the maintenance interval time (oil replacement) in the thesis, probabilistic models of reliability and failure rates could be used to estimate the optimum maintenance interval t_m and the life t_l. A number of studies have been conducted in reliability based methods for estimating the optimum maintenance interval and optimum replacement interval for non-maintained and maintained systems. Some examples of these studies can be found in references [Jard73, Bloc86, Park88, Regu83, Geur83].

Suppose we denote the reliability as a function of time of a system without maintenance as $R(t)$, where t is the operation time of the system. If we perform maintenance on the system at intervals T, then for $t < T$, maintenance will have no effect on reliability. That is, the reliability of the maintained system $R_M(t)$ is given by

$$R_M(t) = R(t), \quad \text{for } 0 \leq t < T. \tag{9.13}$$

After maintenance at time T, in the interval $T \leq t < 2T$, the reliability is the product of the probability $R(t)$ that the system survived to time T, and the probability $R(t\text{-}T)$ that a system as good as new at time T will survive for a time $(t\text{-}T)$ without failure, i.e.

$$R_M(t) = R(T)\mathrm{R}(t-T), \quad \text{for } T \leq t < 2T \tag{9.14}$$

The same argument can be used repeatedly to obtain a general expression for the reliability of the maintained system,

$$R_M = R(T)^n \, T(t-nT), \quad \text{for } nT < t < (n+1)T, \quad n=0,1,2,... \tag{9.15}$$

Thus, mean time to failure (MTTF) for a system with preventive maintenance can be expressed as

$$MTTF = \int_0^\infty R_M(t)dt = \frac{\int_0^\infty R(t)dt}{1-R(T)} = E(t_l) \tag{9.16}$$

Total cost of the system operation, maintenance and failure based replacement can be estimated as

$$C_T(t) = N_m(t)C_m(t) + [1-R_M(t)]\, C_f(t) + C_i(t) + C_o(t), \tag{9.17}$$

where

N_m = number of maintenance operations performed by time t,

$C_m(t)$ = cost of each preventive maintenance of the system,

$C_f(t)$ = cost of failure of the system (includes down time and damage to other systems but not the cost of replacement),

$C_I(t)$ = initial investment in the system (includes the initial cost, cost of installation, and cost of down time for installation),

$C_o(t)$ = other operating costs (in case of bearing costs such as energy loss due to friction, oil cleaning/filtering costs).

Substituting the following expression for operation time t in the above equation

$$t = N_m\, T + t_e, \quad \text{with} \quad t_e \ll t, \quad \text{or } t_e \to 0 \tag{9.18}$$

to find the total cost per unit time of operation,

$$C_{uT}(t,T) = \frac{C_m(t)}{T} + \left[1 - R(T)^{t/T}\right]\frac{C_f}{t} + \frac{C_I(t)}{t} + \frac{C_o(t)}{t} \tag{9.19}$$

Minimization of this quantity can give the optimum preventive maintenance time T_m^* and optimum life t_l^* from the expression for *MTTF* in equation 9.16.

9.3.4 Sensitivity Analysis of Optimum Solution

After finding the optimum solution to life-cycle problem an additional step that can be highly valuable is the sensitivity analysis of optimum solution to problem

parameters (design parameters and design constraints). Such a sensitivity analysis of the optimum solution is useful in order to perform the cost/benefit analysis of investment in R&D and new technologies. An additional application of sensitivity analysis is to provide a basis for formal decomposition of the multi-stage (serial) problem to a multilevel (hierarchical) problem. Following is a brief discussion of the sensitivity analysis problem and its formulation.

Sobieszczanki-Sobieski et al. [Sobi82, Sobi83] give a method of estimating the sensitivity derivatives directly from a set of constrained optimization problems. The equations can be used regardless of the type of optimization algorithm that is used to arrive at the optimum solution. Sobieszczanki-Sobieski et al. [Sobi82] also state the conditions for solvability of the sensitivity equations developed.

There are at least two ways of deriving the equations for the unknown sensitivities. One way is to start from the Lagrange multiplier equations of the constrained minimum; the other method begins with the extremum conditions of the penalty function. In either case, the same general functional relationships are recognized. The first method as described by Sobieszczanki-Sobieski et al. is now briefly presented as applicable to the life-cycle cost optimization problem.

Let the objective function be

$$F(X_1,X_2,\ldots,X_n\ p_1,p_2,\ldots,p_k) \tag{9.20}$$

and the constraints active at the optimum point be

$$g_j(X_1,X_2,\ldots,X_n\ p_1,p_2,\ldots,p_k),\quad \text{for } j=1 \text{ to } m, \tag{9.21}$$

and implicitly at the optimum point $X_i^*(p_1,p_2,..p_k)$. Here $X_1,X_2,..X_n$ are the design variables, and $p_1,p_2,..p_k$ are the parameters of the problem. There is no need to distinguish among the equality and inequality constraints in the optimization problem, because all the constraints active (let it be a set of constraints J) at the constrained optimum point may be regarded as the equality constraints. The Lagrange multiplier equations satisfied at the constrained minimum point are

$$\nabla F(X) + \sum_j \lambda_j \nabla g_j(X) = 0, \quad \text{for } j \in J \tag{9.22}$$

$$g_j(X) = 0, \text{ and } \lambda_j = 0, \quad \text{for } j \in J \tag{9.23}$$

It is understood here that X and λ correspond to their values at the optimum point X^* and λ^*, respectively. Differentiating the first equation with respect to a problem parameter p, we have

$$\sum_{k=1}^{n}\left[\frac{\partial^2 F(X)}{\partial X_i \partial X_k} + \sum_{j=1}^{J}\lambda_j \frac{\partial^2 g_j(X)}{\partial X_i \partial X_k}\right]\frac{\partial X_k}{\partial p} + \sum_{j=1}^{J}\frac{\partial g_j(X)}{\partial X_i}\frac{\partial \lambda_j}{\partial p} + \frac{\partial^2 F(X)}{\partial X_i \partial p} + \sum_{j=1}^{J}\lambda_j \frac{\partial^2 g_j(X)}{\partial X_i \partial p} = 0$$

$$\text{for } j \in J \tag{9.24}$$

Also differentiating the constraint equation with respect to parameter p

$$\sum_{i=1}^{n}\frac{\partial g_j(X)}{\partial X_i}\frac{\partial X_j}{\partial p} + \frac{\partial g_j(X)}{\partial p} = 0 \tag{9.25}$$

$$\text{for } j \in J$$

We can now determine the value of the terms $\partial X/\partial p$, and $\partial \lambda_j/\partial p$ for i=1,..n and $j \in J$, by writing the above two equations in the matrix form

$$\begin{bmatrix} A_{n\times n} & B_{n\times J} \\ B_{J\times n} & O_{J\times J} \end{bmatrix} \begin{bmatrix} \partial X/\partial p \\ \partial \lambda/\partial p \end{bmatrix} + \begin{bmatrix} c_{n\times 1} \\ d_{J\times 1} \end{bmatrix} = \mathbf{0} \tag{9.26}$$

Here the Lagrange multipliers λ^* are either determined as part of optimization process, or they can be determined from

$$\lambda^* = - [\mathbf{B}^T]^{-1}[\mathbf{B}^T]\ \nabla F(X) \tag{9.27}$$

A fundamental requirement here is that active constraints be linearly independent of each other. We can now calculate the derivative of the objective function at the optimum point as follows

$$\frac{dF(X)}{dp} = \frac{\partial F(X)}{\partial p} + \nabla F(X) \cdot \frac{\partial X}{\partial p} \tag{9.28}$$

Thus the sensitivity of the objective function and design variables with respect to parameter p can be determined. The information now available can be used to estimate when a currently active constraint in J will cease to be active or when some new constraint will become active. An equality constraint (in J) which is presently active will become inactive when its Lagrange multiplier goes to zero. Therefore due to a change Δp in the parameter p, Lagrange multiplier changes by $(\partial \lambda_j/\partial p)\Delta p$, hence

$$\lambda_j + (\partial \lambda_j/\partial p)\Delta p = 0, \text{ or, } \Delta p = - \lambda_j/(\partial \lambda_j/\partial p) \tag{9.29}$$

Similarly, a currently inactive constraint will become critical when its value goes to zero, so

$$g_j(X) + [\nabla g_j(X)\cdot\partial X/\partial p]\Delta p = 0, \quad \text{or,} \quad \Delta p = -g_j(X) \,/\, [\nabla g_j(X)\cdot\partial X/\partial p] \qquad (9.30)$$

Thus from the above formulations, the sensitivities of optimum design variable X_j^* and sensitivity of optimum objective function F^* with respect to a parameter p can be obtained. The sensitivity analysis can thus be used for further improving the life-cycle cost of the products.

APPENDIX A
MULTIDIMENSIONAL INTERPOLATION

In the dynamic programming approach, in order to compute the optimum value function for a stage, a general multi-dimensional interpolation scheme is used as described in this appendix. In solving the problem of interpolating a function ϕ which is a function of a N dimensional vector, it is assumed that a grid of nodes in N dimensional space has been laid with the node coordinates and the function value at those node points given. More specifically, let v be a function of a N dimensional vector $\boldsymbol{u}$ defined by some unknown function ϕ,

$$v = \phi(\boldsymbol{u}) \tag{A.1}$$

where

$$\boldsymbol{u} = \{u_1, u_2, \ldots, u_N\}. \tag{A.2}$$

Furthermore a reasonable number of discretizations of the range of possible values for variable u_n and the corresponding values of v are given. Let vectors $\boldsymbol{u}_{min}$ and $\boldsymbol{u}_{max}$ denote the range of possible values of vector $\boldsymbol{u}$ components as,

$$\boldsymbol{u}_{min} = \{u_{1,min}, \ldots, u_{n,min}, \ldots, u_{N,min}\}, \tag{A.3a}$$

and

$$u_{max} = \{u_{1,max}, \ldots, u_{n,max}, \ldots, u_{N,max}\}, \tag{A.3b}$$

such that

$$u_{n,min} \le u_n \le u_{n,max} \tag{A.4}$$

If L_n equally spaced discretizations are done for the component u_n within the range $(u_{n,min}, u_{n,max})$, then the ℓ'th discretization of u_n can be given by

$$\bar{u}_{n,\ell} = u_{n,\min} + \frac{u_{n,\max} - u_{n,\min}}{L_n - 1} (\ell - 1), \quad \ell = 1,\ldots,L_n; \quad n = 1,\ldots,N. \tag{A.5}$$

Let all discretizations of n'th component u_n be stored in vector $\bar{\boldsymbol{u}}_n$ as follows

$$\bar{\boldsymbol{u}}_n = \{\bar{u}_{n,1}, \ldots, \bar{u}_{n,\ell}, \ldots, \bar{u}_{n,Ln}\}, \quad \text{for } n=1,\ldots,N \tag{A.6}$$

which are stored in a $L \times N$ matrix $\bar{\boldsymbol{U}}$ as row vectors as follows

$$\bar{\boldsymbol{U}} = \begin{bmatrix} \bar{\boldsymbol{u}}_1 \\ \cdot \\ \cdot \\ \bar{\boldsymbol{u}}_n \\ \cdot \\ \cdot \\ \bar{\boldsymbol{u}}_N \end{bmatrix} \tag{A.7}$$

The total number of the sets of combinations generated by such discretizations $\bar{\boldsymbol{u}}_n$ is given by

$$K = \prod_{n=1}^{N} L_n \tag{A.8}$$

This means that the discretizations generate a total of K nodal points in N dimensional space. Furthermore let, these K nodal points $\boldsymbol{u}_k$ in N dimensional space be stored in a $K{\times}N$ matrix $\boldsymbol{U}$ with rows containing the nodal points as follows

$$\boldsymbol{U} = \begin{bmatrix} \boldsymbol{u}_1 \\ \cdot \\ \cdot \\ \boldsymbol{u}_k \\ \cdot \\ \cdot \\ \boldsymbol{u}_K \end{bmatrix} \tag{A.9}$$

where

$$\boldsymbol{u}_k = \{u_{1k}..\ u_{nk}..\ u_{Nk}\},\ \text{ for } \ k{=}1,...,K, \ \text{ and } n{=}1,...,N. \tag{A.10}$$

The corresponding function values (at nodal point k) v_k are stored in a vector $\boldsymbol{V}$, such that

$$v_k = \phi(\boldsymbol{u}_k),\ \text{ for } k{=}1,...,K. \tag{A.11}$$

In the interpolation problem encountered in the general dynamic programming problem, matrices $\overline{\boldsymbol{U}}$ and $\boldsymbol{U}$, and vector $\boldsymbol{V}$ are assumed to be given. The objective is to evaluate the unknown function $v{=}\phi(\boldsymbol{u})$ for a given vector u^* using interpolation of surrounding nodal points in N dimensional space. In this thesis, both quadratic and cubic

polynomial interpolations were implemented. We will denote the degree of interpolation (the maximum value of exponent of u_n) by p — value of p being 2 for quadratic interpolation and 3 for cubic interpolation.

Quadratic Interpolation

In the quadratic interpolation, if the vector u is of two dimensions, then we can define a following interpolation function

$$\phi(\boldsymbol{u}) = a_{00} + a_{10}u_1 + a_{01}u_2 + a_{11}u_1u_2 + a_{20}u_1^2 + a_{02}u_2^2$$
$$+ a_{21}u_1^2u_2 + a_{12}u_1u_2^2 + a_{22}u_1^2u_2^2 \qquad \text{(A.12)}$$

The powers of the u_1 and u_2 in quadratic 2 dimensional case given by this equation can be put in a tabular form as in Table A.1. Similarly, when vector $\boldsymbol{u}$ is 3 dimensional, the coefficients of the interpolation polynomial and corresponding exponents can be shown in a tabular form as in Table A.2 and the function $\phi(\boldsymbol{u})$ is given by

$$\phi(\boldsymbol{u}) = \sum_{i=1}^{27} a_{j_1j_2j_3}\, u_1^{j_1}\, u_2^{j_2}\, u_3^{j_3} \qquad \text{(A.13)}$$

Table A.1: Exponents of polynomial terms for 2 dimensional quadratic interpolation.

Polynomial Term Number i	Exponents of u_1 u_2
1	0 0
2	0 1
3	0 2
4	1 0
5	1 1
6	1 2
7	2 0
8	2 1
9	2 2

By induction, for quadratic interpolation, we can conclude the following points about the interpolation polynomial ϕ which is a function of N dimensional vector $\boldsymbol{u}$:

- The polynomial expression for ϕ for quadratic interpolation has $I_N = 3^N$ terms.
- The i'th term of the polynomial has a coefficient 'a_i' of the following form

$$a_i \equiv a_{j_1 j_2 \cdots j_N} \tag{A.14}$$

- The i'th term exponents for N components of vector $\boldsymbol{u}$ are $j_1,\ldots,j_n,\ldots,j_N$ respectively.
- The set $[j_1 \ldots, j_n \ldots, j_N]$ forms a number in base N (binary, tertiary etc.) which is which is related to the term number i as follows,

$$i = [j_1 \ldots j_n \ldots j_N]_{\text{base N}} + 1, \quad \text{for } i=1,\ldots,I$$

Table A.2: Exponents of polynomial for quadratic interpolation in 3 dimensions.

Polynomial Term Number i	Coefficient of i'th Term	Exponents for u_1 u_2 u_3 (j_1 j_2 j_3)
1	a_{000}	0 0 0
2	a_{001}	0 0 1
3	a_{002}	0 0 2
4	a_{010}	0 1 0
5	a_{011}	0 1 1
6	a_{012}	0 1 2
7	a_{020}	0 2 0
8	a_{021}	0 2 1
9	a_{022}	0 2 2
10	a_{100}	1 0 0
11	a_{101}	1 0 1
12	a_{102}	1 0 2
13	a_{110}	1 1 0
14	a_{111}	1 1 1
15	a_{112}	1 1 2
16	a_{120}	1 2 0
17	a_{121}	1 2 1
18	a_{122}	1 2 2
19	a_{200}	2 0 0
20	a_{201}	2 0 1
21	a_{202}	2 0 2
22	a_{210}	2 1 0
23	a_{211}	2 1 1
24	a_{212}	2 1 2
25	a_{220}	2 2 0
26	a221	2 2 1
27	a_{222}	2 2 2

Consider a hexahedron or a brick in N dimensional space (a N-cube) which has 2^N vertices. If it has nodes on all the vertices, on mid-edges, mid faces and at mid -volume, then the total number of such nodes is given by I_N=3^N. In one dimension the nodes are I_1=3, in 2 dimensions I_2=9 and in 3 dimensions I_3=27. In the interpolation problem, the next step is to find a N-cube from a given set of discretizations in $\overline{\boldsymbol{U}}$ such that the point $\boldsymbol{u}^*$ to be interpolated lies inside this N-cube. This is done by searching for a point $\overline{\boldsymbol{u}}^o$ through the given discretizations in $\overline{\boldsymbol{U}}$ such that the n'th component of $\overline{\boldsymbol{u}}^o$ is largest available discretization point but smaller than the n'th component of $\boldsymbol{u}^*$, i.e.

$$\overline{u}_{n,\ell} \leq \overline{u}_{n,\ell no} \leq u^*_n, \tag{A.15a}$$

$$\text{for } \ell \in [1,..L_n],\ \ \ell_{no} \in [1,..L_n], \text{ and } \ell \neq \ell_o \tag{A.15b}$$

This is repeated for all the N components (n=1,..N) of $\overline{\boldsymbol{U}}$ and the indices ℓ_{no} for $\overline{u}_{n,\ell no}$ are found. We will denote the N-cube vertex ($\overline{u}_{n,\ell o}$, n=1,..N) thus found, by $\overline{\boldsymbol{u}}^o$ and all the indices by vector $\{\ell_o\}$. This completes the identification of all the indices ($\ell_{o,1}$,..$\ell_{o,N}$) for one vertex of the cube. The coordinates of the rest of the vertices of N-cube are found by incrementing the indices one at a time. If the i'th vertex of N-cube $\boldsymbol{u}'_i$ is defined as

$$\boldsymbol{u}'_i = \{u'_{i,1},\ldots,u'_{i,n},\ldots,u'_{i,N}\} \tag{A.16}$$

then N indices in $\overline{U}$ of the i'th vertex u'_i are given by

$$u'_{i,n} = \overline{U}_{n,\ell_{i,n}} \tag{A.17}$$

where indices $\ell_{i,n}$ are evaluated using

$$[\ell_i]_{baseN} = [\ell_{i,1},\ldots,\ell_{i,n},\ldots,\ell_{i,N}]_{baseN} = [\ell_{i,0}\ldots,\ell_{0,n},\ldots,\ell_{0,N}]_{baseN} + (i-1)_{baseN} \tag{A.18}$$

Hence the indices in $\overline{U}$ for subsequent vertex i+1 are found by just algebraically adding 1 to the number in base N defined by indices $[\ell_i]_{base\,N} = [\ell_{i,1},\ldots,\ell_{i,n},\ldots,\ell_{i,N}]$. For the vertex i of the N-cube, the corresponding row k in matrix $\overline{U}$, is simply the decimal (base 10) value of $[\ell_i]_{base\,N}$, i.e.

$$k_{base\ 10} \equiv [\ell_{i,1},\ldots,\ell_{i,n},\ldots,\ell_{i,N}]_{base\ N} \tag{A.19}$$

Finally, for quadratic interpolation in N dimensional space, the polynomial for function ϕ is of the following form,

$$\phi(u) = \sum_{i=[j_1\, j_2 \ldots j_N]_{baseN}=1}^{I_N} a_{j_1 j_2 \ldots j_N}\, u_1^{j_1}\, u_2^{j_2},\ldots,u_N^{j_N} \tag{A.20}$$

Here, note that the number of polynomial terms I_N for ϕ is same as the number of nodes defined by the vertices of the N-cube just considered. Hence the polynomial of equation (A.20) can be used for interpolating the given function values v'_i at the vertices u'_i (i=1,..I_N) of the N-cube. This is true since it leads to a set of I_N

simultaneous linear equations in I_N unknown coefficients a_i's. Let the multiplier's at the nodal points u'_i be stored as an element of a $I_N \times I_N$ matrix U'

$$U'_{ii'} = \prod_{n=1}^{N} u'_{i',n}{}^{j_n} \tag{A.21}$$

where i' denotes the polynomial term number in for the equation at i'th node point. Let the corresponding function values at each node of the N-cube v'_i be stored in the column vector V'. Furthermore, let the coefficients a_i of the polynomials for function ϕ be stored in column vector A. Then the substitution of I_N nodal coordinates u'_i and corresponding values v'_i into equation (A.15) results in the following set of I_N simultaneous linear equations in I_N unknowns coefficients

$$\{V'\} = [U']\ \{A\} \tag{A.22}$$

This equation is solved for the polynomial coefficients as follows

$$\{A\} = [U'^{-1}]\ \{V'\} \tag{A.23}$$

Thus the value of the function ϕ at the given point $\{u^*\}$ is obtained as follows

$$v^* = \{u^*\}\cdot\{A\} \tag{A.24}$$

This completes the discussion of quadratic interpolation in N dimensional space.

APPENDIX B
SIMULATION EXAMPLES

In this appendix, two examples of the journal bearing simulations are presented. The first simulation is performed with hydrodynamic model and the second with thermo-hydrodynamic model. The resulting performance parameter during the simulation are plotted for the life of the bearing.

B.1 Simulation 1 — Hydrodynamic Model

For the simulation with hydrodynamic model, the following bearing design and operation parameters are specified.

B.1.1 Input Parameters

Design Parameters

Bearing Diameter, D	1.32 inch
Bearing Speed, N	5000 rpm
Load, W_o	1000 lb
Bearing material	Lead-based Babbitt
Bearing material hardness, H_b	21
Journal material hardness, H_s	200
Contaminant hardness, H_c	100
Bearing wear coefficient, K_b	0.02
Journal wear coefficient, K_s	0.01
Rotor mass, M	5.0 lbm
Oil inlet temperature, T_i	110 °F

Fixed cost of manufacturing, internal quality and external quality (C_{MIEQ})	300.0 \$
Tolerance on bearing diameter, Δ_D	0.0004 in,
Bearing length, L	1.75 in,
Bearing clearance, C_i	0.001 in,
Lubricant oil viscosity at temperature T_i, μ	1.0×10^{-6} Reyn,
(Relative oil viscosity (G_μ)	0.15)

Simulation parameters

Time increment for simulation	10.0 hour

Operation and Support Parameters

Oil cost rate, r_o	0.02 \$/in^3
Energy cost rate, r_e	0.01 \$/kWhr
Total oil volume, V_o	250 in^3
Initial contaminants/oil ratio, $r_{co,i}$	1.0×10^{-8}

Coefficients for evaluating oil viscosity deterioration:

$r_{co,e}$	1.0×10^{-8}
n_o (equation 3.10)	1.2×10^{7} revolutions
t_o (equation 3.11c)	50.0 hours

Design Constraints

The following constraints are used for the determination of (i) the point for oil replacements and, (b) the life of the bearing.

Lower limit on minimum film thickness, $h_{o,min}$	5.0×10^{-5} in
Upper limit on maximum oil pressure, P_{max}	30000 psi
Maximum oil temperature, T_{max}	300 °F
Lower limit on oil viscosity, μ_{min}	1.0×10^{-7} Reyn
Upper limit on oil viscosity, μ_{max}	5.0×10^{-5} Reyn
Upper limit on contaminant/oil ratio, r_{co}	1.0×10^{-7}
Lower limit on L/D ratio, $(L/D)_{min}$	0.25
Upper limit on L/D ratio, $(L/D)_{max}$	1.00
Lower limit on clearance, C_{min}	0.00075 in
Upper limit on clearance, C_{max}	0.02 in
Lund and Saible instability constraint	

B.1.2 Simulation Results

The results of the simulations are plotted in from Figure B.1 through B.16.

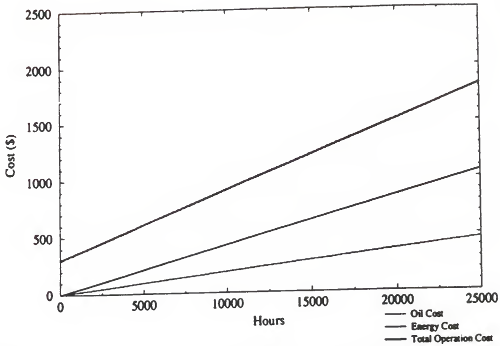


Figure B.1: Oil, energy, and total cost versus hours of operation — simulation with hydrodynamic model.

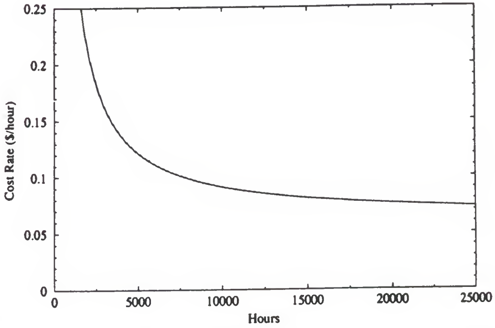


Figure B.2: Cost rate (c_r) versus hours of operation — simulation with hydrodynamic model.

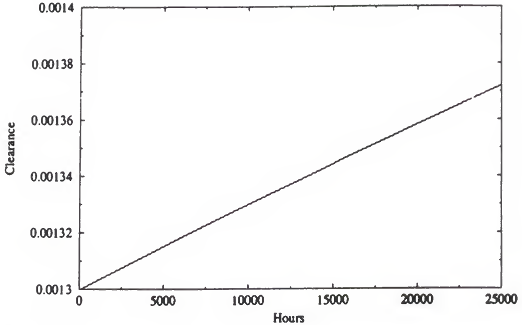


Figure B.3: Clearance (C) versus hours of operation — simulation with hydrodynamic model.

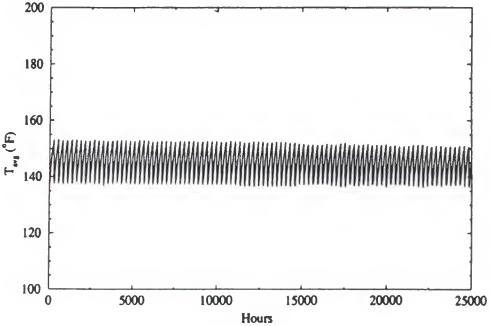


Figure B.4: Average oil temperature (T_{avg}) versus hours of operation — simulation with hydrodynamic model.

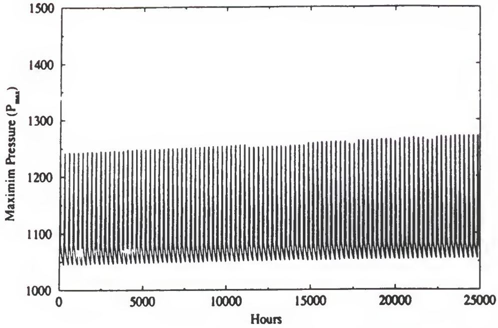


Figure B.5: Maximum oil pressure (P_{max}) versus hours of operation — simulation with hydrodynamic model.

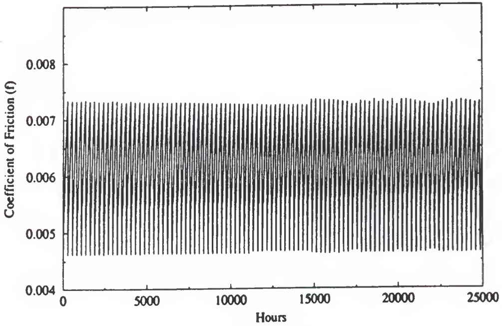


Figure B.6: Coefficient of friction (f) versus hours of operation — simulation with hydrodynamic model.

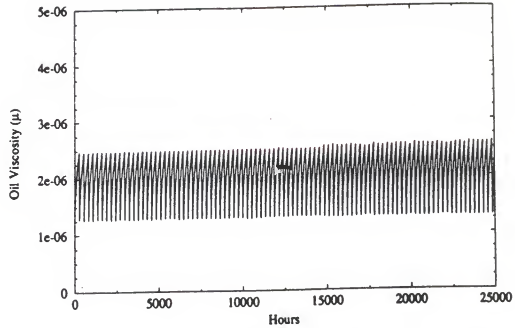


Figure B.7: Oil viscosity (μ) versus hours of operation — simulation with hydrodynamic model.

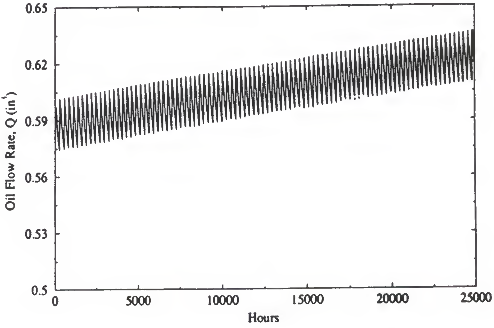


Figure B.8: Oil flow rate (Q) versus hours of operation — simulation with hydrodynamic model.

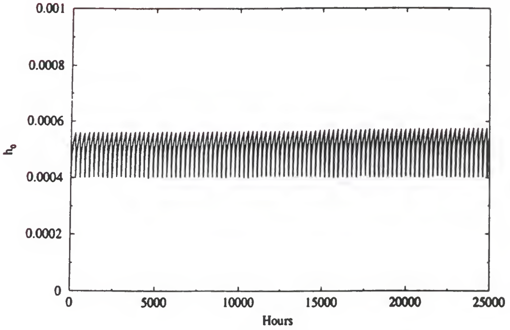


Figure B.9: Minimum oil film thickness (h_o) versus hours of operation — simulation with hydrodynamic model.

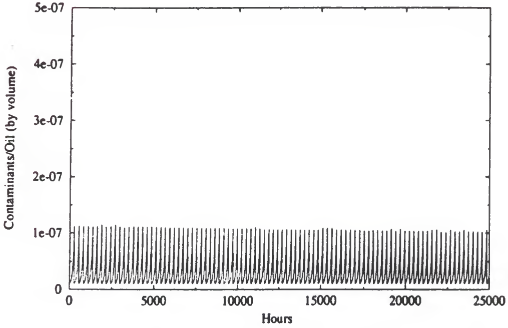


Figure B.10: Contaminants to oil ratio (r_{∞}) versus hours of operation — simulation with hydrodynamic model.

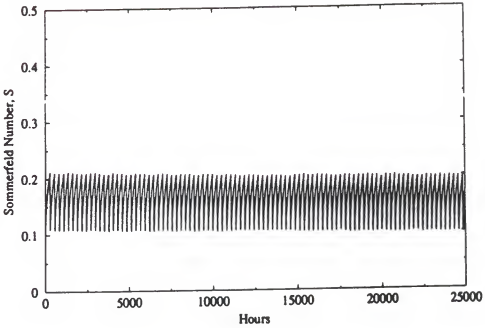


Figure B.11: Sommerfeld number (S) versus hours of operation — simulation with hydrodynamic model.

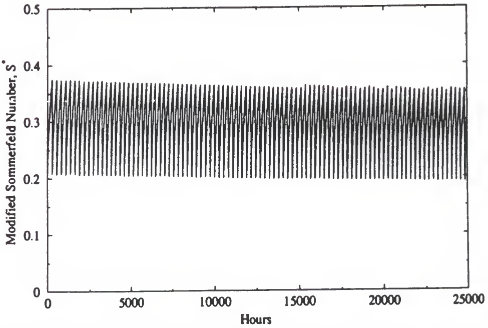


Figure B.12: Modified Sommerfeld number (S^*) versus hours of operation — simulation with hydrodynamic model.

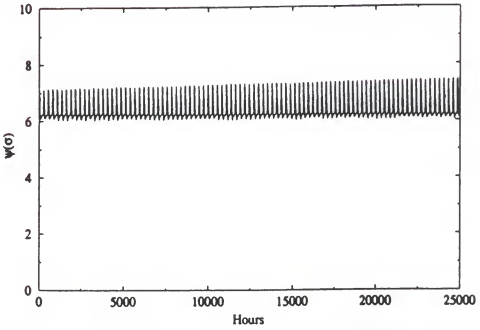


Figure B.13: Lund and Saible stability parameter ($\psi(\sigma)$) versus hours of operation — simulation with hydrodynamic model.

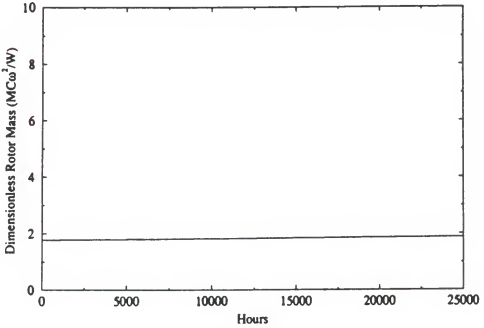


Figure B.14: Dimensionless rotor mass ($MC\omega^2/W$) versus hours of operation — simulation with hydrodynamic model.

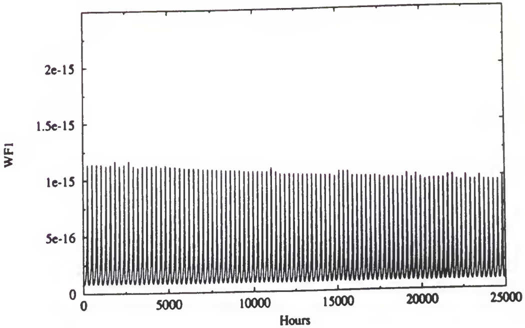


Figure B.15: Wear factor (WF_1) versus hours of operation — simulation with hydrodynamic model.

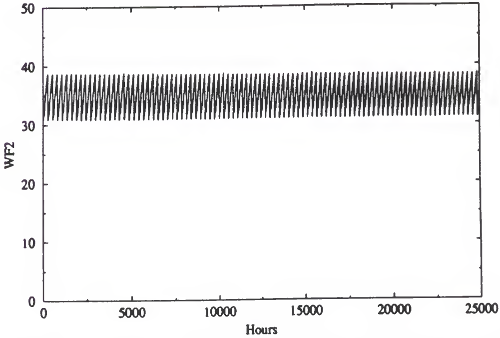


Figure B.16: Wear factor (WF_2) versus hours of operation — simulation with hydrodynamic model.

B.2 Simulation 2 — Thermo-hydrodynamic Model

B.2.1 Input Parameters

The above example was also run with thermo-hydrodynamic model with the following changes.

Initial clearance, C_i	0.0015 in
Relative viscosity, G_μ	0.4
(Oil viscosity μ at temperature T_i	2.688×10^{-6})

B.2.2 Simulation Results

The results of the simulations are plotted in from Figure B.17 through B.32.

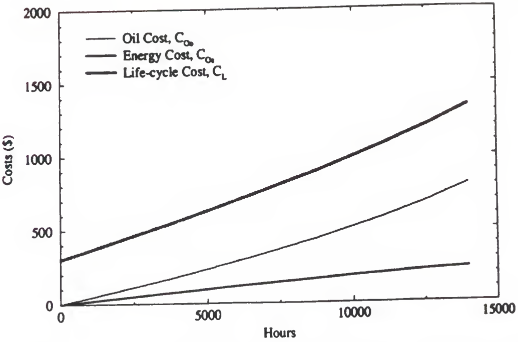


Figure B.17: Oil, energy, and total cost versus hours of operation — simulation with thermo-hydrodynamic model.

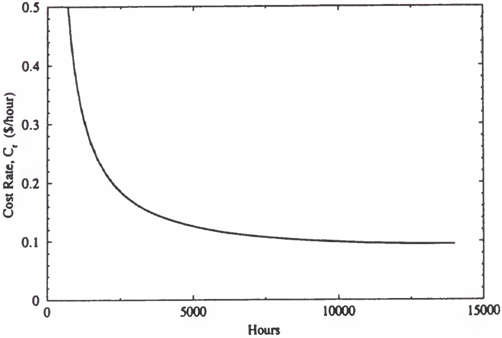


Figure B.18: Cost rate (c_r) versus hours of operation — simulation with thermo-hydrodynamic model.

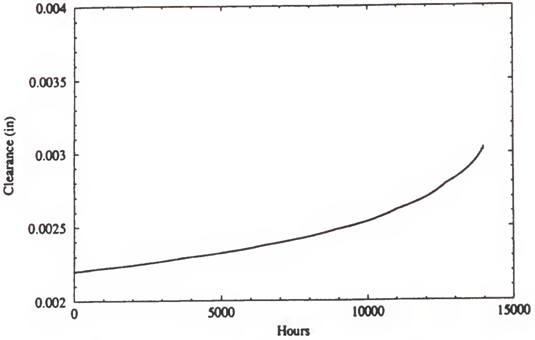


Figure B.19: Clearance (C) versus hours of operation — simulation with thermo-hydrodynamic model.

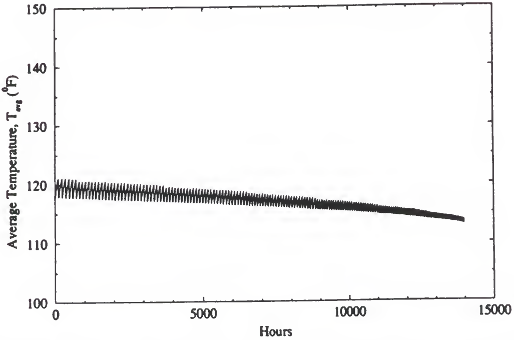


Figure B.20: Average oil temperature (T_{avg}) versus hours of operation — simulation with thermo-hydrodynamic model.

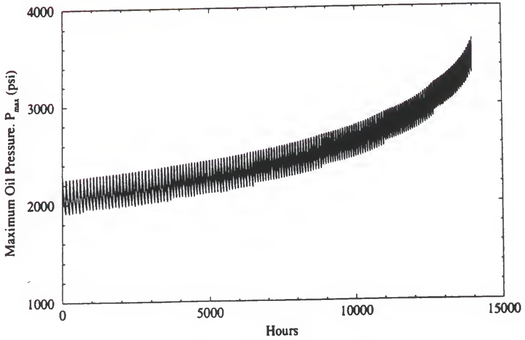


Figure B.21: Maximum oil pressure (P_{max}) versus hours of operation — simulation with hydrodynamic model.

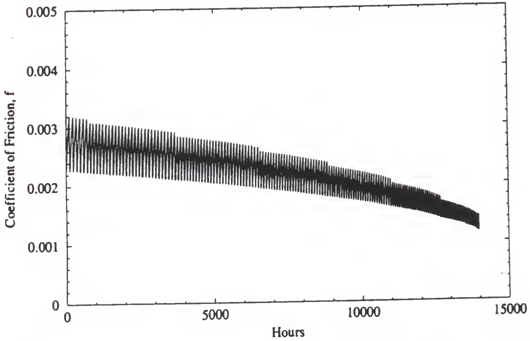


Figure B.22: Coefficient of friction (f) versus hours of operation — simulation with thermo-hydrodynamic model.

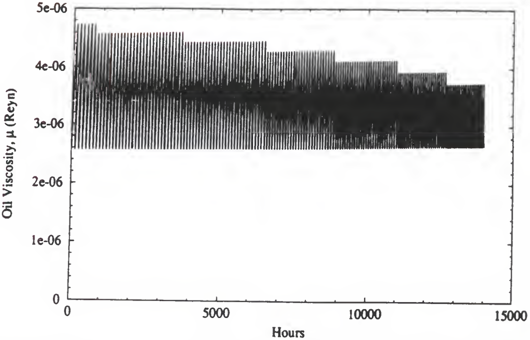


Figure B.23: Oil viscosity (μ) versus hours of operation — simulation with thermo-hydrodynamic model.

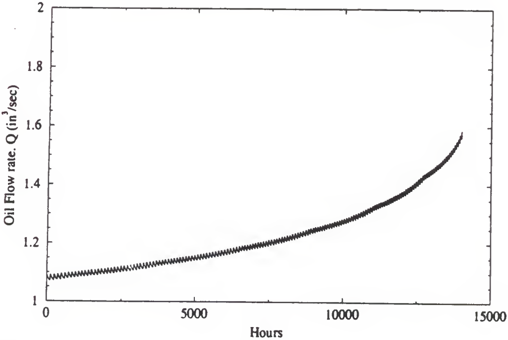


Figure B.24: Oil flow rate (Q) versus hours of operation — simulation with thermo-hydrodynamic model.

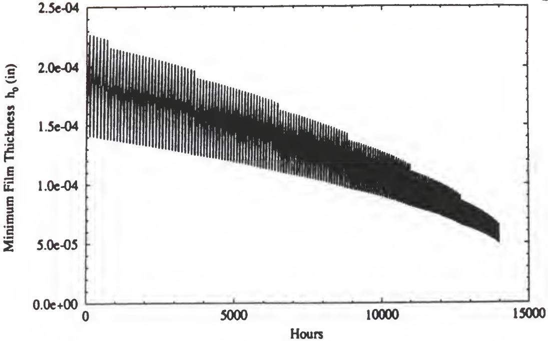


Figure B.25: Minimum oil film thickness (h_o) versus hours of operation — simulation with thermo-hydrodynamic model.

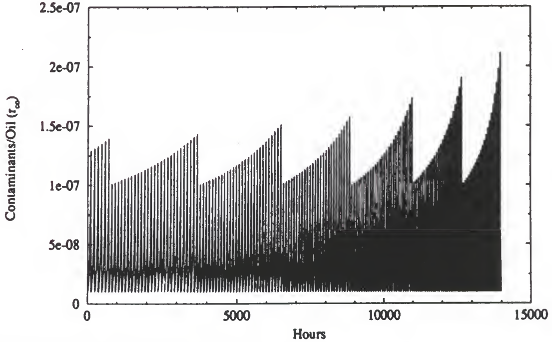


Figure B.26: Contaminants to oil ratio (r_{co}) versus hours of operation — simulation with thermo-hydrodynamic model.

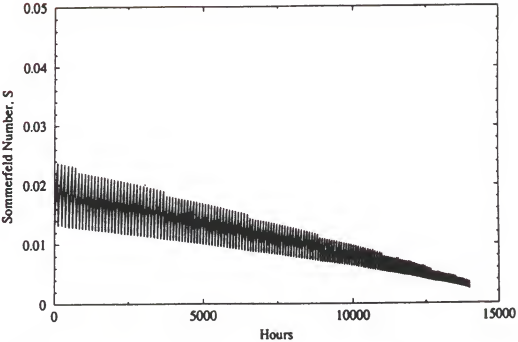


Figure B.27: Sommerfeld number (S) versus hours of operation — simulation with thermo-hydrodynamic model.

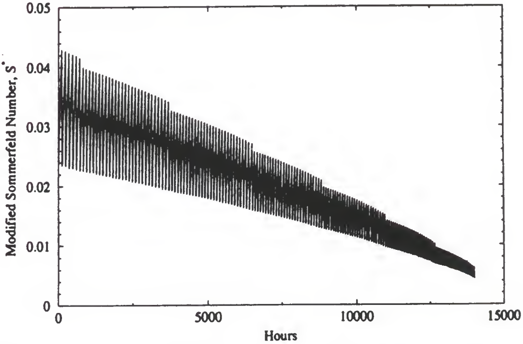


Figure B.28: Modified Sommerfeld number (S^*) versus hours of operation — simulation with thermo-hydrodynamic model.

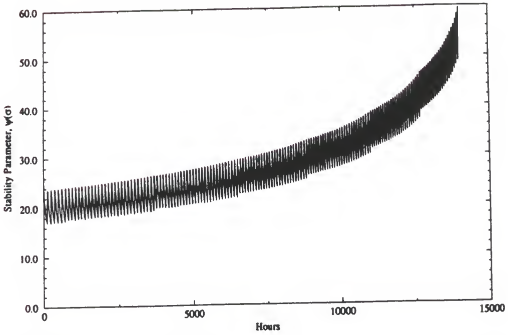


Figure B.29: Lund and Saible stability parameter ($\psi(\sigma)$) versus hours of operation — simulation with thermo-hydrodynamic model.

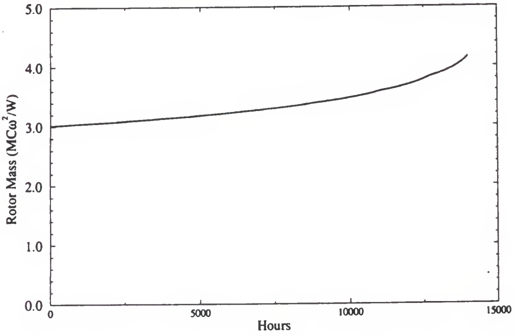


Figure B.30: Dimensionless rotor mass ($MC\omega^2/W$) versus hours of operation — simulation with thermo-hydrodynamic model.

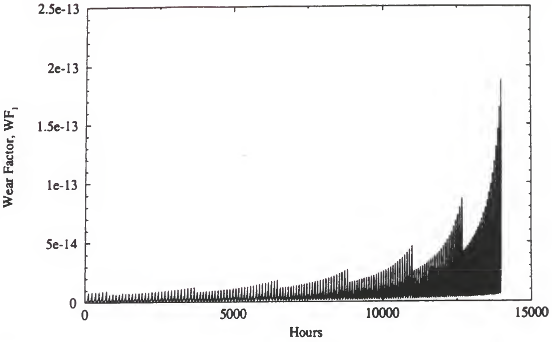


Figure B.31: Wear factor (WF_1) versus hours of operation — simulation with thermo-hydrodynamic model.

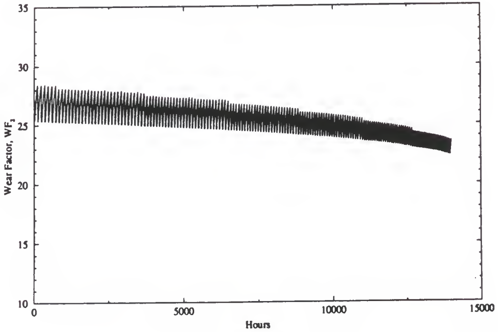


Figure B.32: Wear factor (WF_2) versus hours of operation — simulation with hydrodynamic model.

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